DEPARTMENT OF MECHANICAL ENGINEERING
ME3351 ENGINEERING MECHANICS

UNIT-1- BASICS AND STATICS OF PARTICLES

1ntroduction – Units and Dimensions

Engineering mechanics is that branch of science which deals with the behavior of a body when the body is at rest or in motion. It is mainly divided into statics and dynamics. The goal of this Engineering Mechanics course is to solve problems in mechanics as applied to plausibly real-world scenarios. Mechanics is the study of forces that act on bodies and the resultant motion that those bodies experience. With roots in physics and mathematics, Engineering Mechanics is the basis of all the mechanical sciences: civil engineering, materials science and engineering, mechanical engineering and aeronautical and aerospace engineering.

Laws of Mechanics Lame's theorem, Parallelogram and triangular Law of forces

The parallelogram of forces is a method to determine (or visualizing) the resultant of applying two forces to an object. It states, if two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point. Let P and Q be the force then resultant is R.

If there forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces. Suppose the three forces a, b and c are acting at a point and they are in equilibrium then equation is written as

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$

If two forces are acting simultaneously ona particle and can be represented by the two sides of a triangle taken in order, then the third side represents the resultant in the opposite order. Triangle Law of Vector Addition is great method to add two vectors.Addition of three vectors using triangle Law and then the method is used to demonstrate the polygon law of vector addition.

Vectors- Vectorial representation of forces and moments- Vector operations: additions,subtraction, dot product, cross product

A quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another.For example displacement, velocity, acceleration, force, moment and momentum.A quantity or phenomenon that exhibits magnitude only, with no specific direction, is called a scalar. Examples of scalars include speed, mass, electrical resistance, and hard-drive storage capacity.

"A unit vector is defined as a vector in any specified direction whose magnitude is unity i.e. 1. A unit vector only specifies the direction of a given vector. "A unit vector is denoted by any small letter with a symbol of arrow hat .A unit vector can be determined by dividing the vector by its magnitude.Avector that indicates the position of a point in a coordinate system is referred to as position vector.

Suppose we have a fixed reference point O, then we can specify the position of a given point P with respect to point O by means of a vector having magnitude and direction represented by a directed line segment OP .This vector is called position vector.

Coplanar Forces – Resolution and Composition of forces

When several forces act on a body, then they are called a force system or system of forces. It is necessary to study the system of forces, to find out the net effort of forces on the body. Let us consider a wooden block resting on a smooth inclined plane. It is supported by a force P. the force system for this block consist of weight of the block, reaction of the block on the inclined plane and applied force.

Force system mainly divided into two;

1. Coplanar

a. collinear b. concurrent c. parallel d. non concurrent non parallel

2. Non-coplanar

a. concurrent b. parallel c. non concurrent non parallel

Equivalent systems of forcesPrinciple of transmissibility – Single equivalent force

Steps to find equivalent system:

- Replacing two forces acting at a point by their resultant.
- Resolving the force into two components.
- Cancelling two equal and opposite forces acting at a point.
- Attaching two equal and opposite forces at a point.
- Transmitting a force along its line of action.

Principle of transmissibility states that " the state of rest or motion of a rigid body is unaltered if a force acting on a body is replaced by another force of same magnitude and direction, but acting anywhere on the body along the line of action of the force". A couple is two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

Part –A (2MARKS)

1. What is engineering mechanics?

Engineering mechanics is that branch of science which deals with the behavior of a body when the body is at rest or in motion. It is mainly divided into statics and dynamics. The goal of this Engineering Mechanics course is to solve problems in mechanics as applied to plausibly real-world scenarios. Mechanics is the study of forces that act on bodies and the resultant motion that those bodies experience. With roots in physics and mathematics, Engineering Mechanics is the basis of all the mechanical sciences: civil engineering, materials science and engineering, mechanical engineering and aeronautical and aerospace engineering.

2. Write the application of engineering mechanics.

Engineering mechanics has applications in many areas of engineering projects. To cite a few examples, it is applied in the design of spacecraft's and rockets, analysis of structural stability and machine strength, vibrations, robotics, electrical machines, fluid flow and automatic control. As the oldest physical science, mechanics has very wide application in a variety of engineering fields.We human cannot stand without the Equilibrium; it is also applied to those inventions like cars, appliances, buildings, even in shoes and in a single paper.

3. Define the statics.

The branch of science which deals with the study of a body when body is at rest. Statics is the branch of mechanics that is concerned with the analysis of loads (force and torque, or "moment") on physical systems in static equilibrium, that is, in a state where the relative positions of subsystems do not vary over time, or where components and structures are at a constant velocity. When in static equilibrium, the system is either at rest, or its center of mass moves at constant velocity.

4. Define dynamics.

The branch of science which deals with the study of a body when body is in motion. Dynamics is a branch of physics (specifically classical mechanics) concerned with the study of forces and torques and their effect on motion, as opposed to kinematics, which studies the motion of objects without reference to its causes. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion.

5. What is kinematics?

Kinematics is the branch of classical mechanics which describes the motion of points, bodies (objects) and systems of bodies (groups of objects) without consideration of the causes of motion. To describe motion, kinematics studies the trajectories of points, lines and other geometric objects and their differential properties such as velocity and acceleration. Kinematics is used in astrophysics to describe the motion of celestial bodies and systems, and in mechanical engineering, robotics and biomechanics to describe the motion of systems composed of joined parts (multi-link systems) such as an engine, a robotic arm or the skeleton of the human body.

6. Write about kinetics.

In physics and engineering, kinetics is a term for the branch of classical mechanics that is concerned with the relationship between the motion of bodies and its causes, namely forces and torques. Since the mid-20th century, the term "dynamics" (or "analytical dynamics") has largely superseded "kinetics" in physics text books; the term "kinetics" is still used in engineering. In mechanics, the Kinetics is deduced from Kinematics by the introduction of the concept of mass.

7. Write the importance of engineering mechanics.

The science of engineering mechanics is mainly concerned with the knowledge of state of rest or motion of bodies under the action of different forces. It is common experience that various bodies in the universe are either at rest or in motion. Naturally, to study the behavior of different bodies, the knowledge of engineering mechanics is a paramount importance, so as to execute the design and construction in the engineering field.

Also, basic principles of mechanics are used in the study of subjects such as strength of materials, stability of structures, vibrations, fluid flow, electrical machines etc.

8. What is mechanics of solids?

Solid mechanics is the branch of continuum mechanics that studies the behavior of solid materials, especially their motion and deformation under the action of forces, temperature changes, phase changes, and other external or internal agents.

Solid mechanics is fundamental for civil, aerospace, nuclear, and mechanical engineering, for geology, and for many branches of physics such as materials science. It has specific applications in many other areas, such as understanding the anatomy of living beings, and the design of dental prostheses and surgical implants.

9. What is mechanics of fluids?

Fluid mechanics is the branch of physics which involves the study of fluids (liquids, gases, and plasmas) and the forces on them. Fluid mechanics can be divided into fluid statics, the study of fluids at rest; and fluid dynamics, the study of the effect of forces on fluid motion. It is a branch of continuum mechanics, a subject which models matter without using the information that it is made out of atoms; that is, it models matter from a macroscopic viewpoint rather than from microscopic. Fluid mechanics, especially fluid dynamics, is an active field of research with many problems that are partly or wholly unsolved.

10. Define rigid bodies.

In physics, a rigid body is an idealization of a solid body in which deformation is neglected. In other words, the distance between any two given points of a rigid body remains constant in time regardless of external forces exerted on it. Even though such an object cannot physically exist due to relativity, objects can normally be assumed to be perfectly rigid if they are not moving near the speed of light.

11. Define deformable bodies.

Alteration in the shape or dimensions of an object as a result of the application of stress to it.Any alteration of shape or dimensions of a body caused by stresses, thermal expansion or contraction, chemical or metallurgical transformations, or shrinkage and expansions due to moisture change is called deformable body. In fact all the body will go for deformation when it is loaded. There is no rigid body in actual practice.

12. What is particle?

A particle is defined as a very small quantity of matter that may be considered to occupy a single point in space. Or, a particle is defined as a body whose dimensions are negligible and whose mass is concentrated at a point. Whether objects can be considered particles depends on the scale of the context; if an object's own size is small or negligible, or if geometrical properties and structure are irrelevant, then it can often be considered a particle. For example, grains of sand on a beach can be considered particles because the size of one grain of sand (~1 mm) is negligible compared to the beach, and the features of individual grains of sand are usually irrelevant to the problem at hand

Laws of Mechanics Lame's theorem, Parallelogram and triangular Law of forces

13. State Newton's first law of motion.

It states, Everybody continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by some external force acting on it. When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by an external force. It is also defined as, if the net force (the vector sum of all forces acting on an object) is zero, then the velocity of the object is constant.

14. State Newton's second law of motion.

Newton's second law of motion - the rate of change of momentum is proportional to the imposed force and goes in the direction of the force. Newton's second law of motion can be formally stated as follows: The acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass of the object.

15. State Newton's third law of motion.

For every action, there is an equal and opposite reaction.When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.This law is familiar in everyday situations; a force cannot be applied to an object unless something resists the reaction of that force. In order to walk across the floor, you must push back on thefloor with your foot; then, according to Newton's Third Law, the floor pushes forward on your foot, which propels you forward.

16. State Parallelogram law of forces.

The parallelogram of forces is a method to determine (or visualizing) the resultant of applying two forces to an object. It states, if two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point. Let P and Q be the force then resultant is R.

17. Write about sine law.

Let consider triangle with sides a,b,c and included angle α, β, γ then sine law is written as follows

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$

In trigonometry, the law of sines, sine law, sine formula, or sine rule is an equation, relating the lengths of the sides of any shaped triangle to the sines of its angles.

18. Write about cosine law.

Let consider triangle with sides a, b and c and included angle α, β, γ then cosine law is written as follows

$$a^2 = b^2+c^2-2bc\ \cos\alpha$$
$$b^2 = a^2+c^2-2ac\ \cos\beta$$
$$c^2 = a^2+b^2-2ab\ \cos\gamma$$

19. <u>State and briefly explain Lami's theorem. (May/June 2012)</u>

If there forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces. Suppose the three forces a, b and c are acting at a point and they are in equilibrium then equation is written as

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$

20. Write about triangle law of forces.

If two forces are acting simultaneously ona particle and can be represented by the two sides of a triangle taken in order, then the third side represents the resultant in the opposite order. Triangle Law of Vector Addition is great method to add two vectors.Addition of three vectors using triangle Law and then the method is used to demonstrate the polygon law of vector addition.

21. Write about of gravitational law of attraction.

Newton's Universal Law of Gravitation states that any two objects exert a gravitational force of attraction on each other. The direction of the force is along the line joining the objects. The magnitude of the force is proportional to the product of the gravitational masses of the objects, and inversely proportional to the square of the distance between them.

$$F=G\,\frac{m1\,m2}{r2}$$

Vectors- Vectorial representation of forces and moments- Vector operations: additions,subtraction, dot product, cross product

22. Define vectors.

A quantity having direction as well as magnitude, especially as determining the position of one point in space relative to another.For example displacement, velocity, acceleration, force, moment and momentum.A quantity or phenomenon that exhibits magnitude only, with no specific direction, is called a scalar. Examples of scalars include speed, mass, electrical resistance, and hard-drive storage capacity.

23. What is unit vector and position vector?

"A unit vector is defined as a vector in any specified direction whose magnitude is unity i.e. 1. A unit vector only specifies the direction of a given vector. "A unit vector is denoted by any small letter with a symbol of arrow hat .A unit vector can be determined by dividing the vector by its magnitude.Avector that indicates the position of a point in a coordinate system is referred to as position vector.

Suppose we have a fixed reference point O, then we can specify the position of a given point P with respect to point O by means of a vector having magnitude and direction represented by a directed line segment OP .This vector is called position vector.

24. <u>A force of magnitude 750N is directed along AB where A is (0.8,0,1.2)m and B is (1.4,1.2,0). write the vector form of the force.(Nov/Dec 2003)</u>

$$\vec{B}\vec{A}=0.6\vec{i}+102\vec{j}-1.2\vec{k}$$

$$\text{Unit vector}=\frac{0.6\vec{i}+102\vec{j}-1.2\vec{k}}{\sqrt{0.6^2+1.2^2+1.2^2}}$$

$$=0.333\vec{i}+0.666\vec{j}-0.666\vec{k}$$

$$\bar{F}=750\left(0.333\vec{i}+0.666\vec{j}-0.666\vec{k}\right)$$

$$\bar{F}=250\vec{i}+500\vec{j}-500\vec{k}$$

25. <u>A force $F=10\vec{i}+8\vec{j}-5\vec{z}$ N acts at the point A is (2,5,6)m. what is the moment of the force about the point B (3,1,4)m? (Nov/Dec 2002)</u>

Solution:

$$M_B=\vec{r}\times\vec{F}$$

$$r=(2-3)\vec{i}+(5-1)\vec{j}+(6-4)\vec{k}$$

$$r=-\vec{i}+4\vec{j}+2\vec{k}$$

$$M_B=\begin{vmatrix} i & j & k \\ -1 & 4 & 2 \\ 10 & 8 & -5 \end{vmatrix}$$

$$=-36\vec{i}+15\vec{j}-48\vec{k}$$

26. <u>Two vector A and B are given. Determine their cross product and the unit vector along it. A= 2i+3j+k and B= 3i-3j+4k.(Jan 2003)</u>

Solution

$$\vec{A} \text{ X } \vec{B} = \left(A_X \vec{i} + A_y \vec{j} + A_z \vec{k}\right) \times \left(B_X \vec{i} + B_y \vec{j} + B_z \vec{k}\right)$$

$$= \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 3 & -3 & 4 \end{vmatrix}$$

$$= 15\vec{i} - 5\vec{j} - 15\vec{k}$$

$$\left|\vec{C}\right| = \sqrt{15^2 + 5^2 + 15^2}$$

$$= 21.795$$

$$\text{Unit vector} = \frac{15\vec{i} - 5\vec{j} - 15\vec{k}}{21.795}$$

$$= .688\vec{i} - 0.229\vec{j} - 0.688\vec{k}$$

27. Find the unit vector of a factor $\vec{F} = \vec{4i} - \vec{5j} + \vec{8k}$

$$Unit \text{ vector} \quad \lambda = \frac{\overline{F}}{\left|\overline{F}\right|}$$

$$\left|\overline{F}\right| = \sqrt{4^2 + (-5)^2 + 8^2} = \sqrt{105} = 10.247$$

$$\lambda = \frac{\vec{4i} - \vec{5j} + \vec{8k}}{10.247} = 0.39\vec{i} - 0.49\vec{j} + 0.78\vec{k}$$

28. Define moment of forces.

A moment of force is the product of a force and its distance from an axis, which causes rotation about that axis. In principle, any physical quantity can be combined with a distance to produce a moment; commonly used quantities include forces, masses, and electric charge distributions. Unit of moment is N-m

Moment= Force x Perpendicular distance

29. Define couple.

Two parallel, non-collinear forces of equal magnitude having opposite sense are said t form a couple. Let us consider two forces F and –F of equal magnitude acting in opposite directions whose lines of action are parallel and separated by a distance d. d is the perpendicular distance between the lines of action of forces F and –F. The sum of the components of these two forces in any direction is zero. The combined moment of of these two forces about an axis perpendicular to the plane passing through any point like O is not zero. In other words, the couple has tendency to rotate or turn the body.

30. What is the effect of force and moment on a body?

A force acting on an object may cause the object to change shape, to start moving, to stop moving, to accelerate or decelerate. When two objects interact with each other they exert a force on each other, the forces are equal in size but opposite in direction.A moment of force is the product of a force and its distance from an axis, which causes rotation of object about that axis. It may be clockwise or anti clockwise direction.

31. Write the varignon's theorem.

Principle of moment states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point. The Varignon'stheorem states that the moment of a resultant of two concurrent forces about any point is equal to the algebraic sum of the moments of its components about the same point. It is used to solve the problems of beams.

32. Write about resultant of two parallel forces.

The forces acting on an object can be replaced with a single force that causes the object to behave in the same way as all the separate forces acting together did, this one overall force is called the resultant force.

If the resultant force acting on an object is ZERO then the object will remain stationary. If it was stationary when the resultant force became zero or move at a constant (steady) speed in a straight line if it was moving when the resultant force became zero.

33. <u>Find the magnitude of the resultant of the two concurrent forces of magnitude 60kN and 40kN with an included angle of 70^0 between them. (Apr/May 2011)</u>

Given:

P= 60kN

Q= 40kN

$\theta = 70^0$

To find:

Resultant, R

Solution:

By parallelogram law of forces,

$$R=\sqrt{P^2+Q^2+2PQ}\cos\theta$$

$$R=\sqrt{60^2+40^2+2X60X40}\cos 70$$

R= 82.7 kN

34. Define resolution of a force.

Resolution of forces means finding the components of a given force in two directions. Let force p makes θ with X axis; it is required to find the components of the force P along X axis and Y axis. Component of P along X axis is $P\cos\theta$ and along Y axis $P\sin\theta$. That is single force can be resolved into two components - one directed upwards or downwards and the other directed rightwards or leftwards.

35. Write the resolution of a force into a force and a couple.

A given force F applied to a body at any point A can always be replaced by an equal force applied at another point B together with a couple which will be equivalent to the original force. In mechanics, a couple is a system of forces with a resultant (net or sum) moment but no resultant force. A better term is force couple or pure moment. Its effect is to create rotation without translation, or more generally without any acceleration of the centre of mass. In rigid body mechanics, force couples are free vectors, meaning their effects on a body are independent of the point of application. The resultant moment of a couple is called a torque.

<u>Coplanar Forces – Resolution and Composition of forces</u>

36. What is meant by system of forces?

When several forces act on a body, then they are called a force system or system of forces. It is necessary to study the system of forces, to find out the net effort of forces on the body. Let us consider a wooden block resting on a smooth inclined plane. It is supported by a force P. the force system for this block consist of weight of the block, reaction of the block on the inclined plane and applied force.

37. Write the classification of force system.

Force system mainly divided into two;

1. Coplanar

a. collinear b. concurrent c. parallel d. non concurrent non parallel

2. Non-coplanar

a. concurrent b. parallel c. non concurrent non parallel

38. <u>Define coplanar force.(May/June 2010)</u>

If all the forces in a system lie in a single plane then they are "coplanar forces."

There are different types of coplanar forces.They are:

a.Coplanar Concurrent Forces.

b.Coplanar Non-Concurrent Forces.

c.Coplanar Parallel Forces.

(i) Coplanar like Forces.

(ii) Coplanar unlike Forces.

39. <u>What is coplanar collinear system of forces? .(May/June 2010)</u>

If line of action of all forces acting in a single line then they are" coplanarCollinear Force System".Let forces F1, F2 and F3 acting in a plane in the same line i.e., common line of action then the system of forces is known as coplanar collinear force system. Hence in coplanar collinear system of forces, all the forces act in the same plane and a common line of action.

40. Define coplanar concurrent system of forces.

Line of action of all forces passes through a single point and forces lie in a single plane then they are “coplanarconcurrent forces".Let forces F1, F2 and F3 acting in a plane and these forces intersect or meet at a common point O. this system of forces is known as coplanar concurrent force system. Hence in a coplanar concurrent system of forces, all the forces act in the same plane and they intersect at a common point.

41. Write about coplanar parallel system of forces.

If all the forces are parallel to each other and lie in single plane then they are "coplanar parallel forces".

(i) Coplanar like Parallel Forces:All forces are parallel to each other and lie in a single plan and are action in the same direction.

(ii) Coplanar unlike parallel forces:All forces are parallel to each other and lie in single plane butacting in opposite direction.

42. Define coplanar non-concurrent non-prallel system of forces.

When the forces of their lines of action are not meet at a point and not parallel if the forces lie in the same plane, they are known as coplanar non concurrent non-parallel forces. This consists of a number of vectors that do not meet at a single point and none of them are parallel. These systems are essentially a jumble of forces and take considerable care to resolve.

43. <u>What is non-coplanar force system? (Nov/Dec 2011)</u>

All the forces do not lie in the single plane then the force system is called non-coplanar force system. All forces do not lie in a single plane and line of action does not pass through single point, then they are "non-coplanar non-concurrent forces". All forces do not lie in same plane but line of action passes through single point, and then they are "non-coplanar concurrent forces". All forces are parallel to each other but not lie in single plane then they are "non-coplanar parallel forces".

44. What are resultant forces?

The resultant of a force system is the Force which produces same effect as the combined forces of the force system would do. So if we replace all the combined forces of the force system would do. So if we replace all components of the force by the resultant force, then there will be no change in effect. Hence a single force which can replace a number of forces acting on a rigid body, without causing any change in the external effects on the body, is known as the resultant force.

Equilibrium of a particle -Forces in space – Equilibrium of a particle in space

45. Write about the principle of equilibrium.(Nov/Dec 2011)

The principle of equilibrium states that, a stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces is zero and also the algebraic sum of moments of all the external forces about any point on or off the body is zero.

Mathematically it is expressed by the equations

$$\in F = 0$$
$$\in M = 0$$
$$\in - itrepresentsalgebraicsum$$

46. Write the condition for equilibrium of non-concurrent forces system.

1. The algebraic, sums of the components of the forces along each of two lines at right angles to each other equal zero when forces are not intersecting at same point.
2. The algebraic sum of the moments of the forces about any origin equals zero for above force system.

$$\in F = 0$$
$$\in M = 0$$

47. Write the condition for equilibrium of coplanar concurrent forces system.

1. The algebraic, sums of the components of the forces along each of two lines at right angles to each other equal zero when forces are intersecting at same point.
2. The algebraic sum of the moments of the forces about any origin equals zero.

$$\in F = 0$$
$$\in M = 0$$

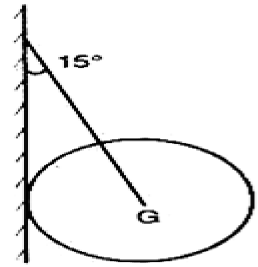
48. What is free body diagram?

Free body diagram of a body can be drawn by removal of all the supports (like wall, hinge, floor or any other body) and replace them by equal reactions which these supports applies on the body. The consideration of internal forces and external forces is essential part.A free body diagram, sometimes called a force diagram, is a pictorial device, often a rough working sketch, used by engineers and physicists to analyze the forces and moments acting on a body.

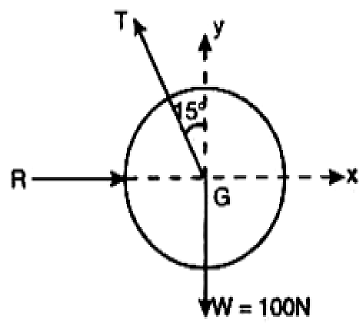
49. Write the steps involved in drawing a free body diagram.(Nov/Dec 2011)

1. Draw a diagram of the body completely isolated from all other bodies
2. Represent the action of each body or support that has been removed by suitable force.
3. Indicate known applied load by its magnitude and unknown load by a symbol.
4. Indicate the weight of the free body with a vertical downward arrow.

50. Draw the free body diagram for given figure.



Solution



51. Define stable equilibrium.

A state in which a body tends to return to its original position after being disturbed, the body said to be stable equilibrium. This condition arises when some additional force sets up due to displacement which brings the body back to its original position.When the center of gravity of a body lies below point of suspension or support, the body is said to be in stable equilibrium. For example a book lying on a table is in stable equilibrium.A book lying on a horizontal surface is an example of stable equilibrium. If the book is lifted from one edge and then allowed to fall, it will come back to its original position.

52. Define unstable equilibrium.

If a body does not return back to its original position and moves farther apart after being slightly displaced from its rest position, the body is said to be in unstable equilibrium. This condition arises when the additional force causes the body to move apart from its rest position. A state of equilibrium of a body (as a pendulum standing directly upward from its point of support) such that when the body is slightly displaced it departs further from the original position

53. Define neutral equilibrium.

When the center of gravity of a body lies at the point of suspension or support, the body is said to be in neutral equilibrium. Example: rolling ball.If a ball is pushed slightly to roll, it will neither come back to its original nor it will roll forward rather it will remain at rest. If the ball is rolled, its center of gravity is neither raised nor lowered. This means that its center of gravity is at the same height as before.

54. What is the difference between a resultant force and equilibrium force? (May/June 2012)

Resultant force: if a number of forces acting simultaneously on a particle, then these forces can be replaced by a single force which would produce the same effect as produced by all forces. This single force is called as resultant force.

Equilibrant force: the force which brings the set of forces in equilibrium is known as equilibrant force. It is equal in magnitude and opposite in direction of the resultant force.

55. What is resultant of concurrent forces in space?

Resultant of a force system is a force or a couple that will have the same effect to the body, both in translation and rotation, if all the forces are removed and replaced by the resultant. Let Rx, Ry and Rz are sum of force components in x, y and z directions then the resultant of tthis system is

$$R=\sqrt{(Rx^2 + Ry^2} + Rz^2)$$

56. Define equilibrium in space.

A rigid body is said to be in equilibrium when the external forces (active and reactive too) acting on it forms a system equivalent to zero. A particle subjected to concurrent force system in space is said to be in equilibrium when the resultant force is zero. In other words,

$\vec{R}=Rx\vec{\imath}=Ry\vec{\jmath}$ = Rz$\vec{k}$ =0

Rx=0; Ry=0; Rz=0 hence the equations of equilibrium for a particle when subjected to concurrent forces in space can be written as

$$\in Fx = 0 \ \in Fy = 0 \ \in Fz = 0$$

Equivalent systems of forcesPrinciple of transmissibility – Single equivalent force

57. What is an equivalent system of forces?

An equivalent system for a given system of coplanar forces is a combination of a force passing through a given point and a moment about that point. The force is the resultant of all forces acting on the body. And the moment is the sum of all the moments about that point.

Hence the equivalent system consists of:

(i) A single force R passing through the given point P and

(ii) A single moment M_R

58. What is meant by force couple system? (May/June 2013)

When a number of forces and couples are acting on a body, they combined into a single force and couple having the same effect. This is called force couple system. It is also termed a equivalent systems. "The condition of equilibrium or motion of a rigid body is remaining unchanged, if force acting on the rigid body is replaced by another force of the same magnitude and same direction but, acting anywhere along the same line of action."

59. Write the steps to find equivalent system.

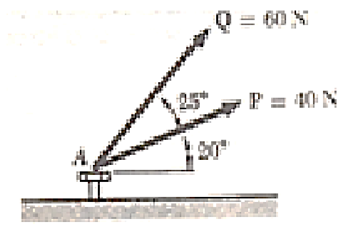
- Replacing two forces acting at a point by their resultant.
- Resolving the force into two components.
- Cancelling two equal and opposite forces acting at a point.
- Attaching two equal and opposite forces at a point.
- Transmitting a force along its line of action.

60. Define transmissibility of forces. (Nov/Dec 2008)

It states that " the state of rest or motion of a rigid body is unaltered if a force acting on a body is replaced by another force of same magnitude and direction, but acting anywhere on the body along the line of action of the force". A couple is two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

<u>**Part-B**</u>

1.(a). The two forces P and Q act on bolt A. Determine their resultant. (8)(May/June 2012)



<u>Solution:</u>

(i) <u>Law Of Cosine:</u>

$$R^2 = P^2 + Q^2 - 2PQ\ \mathrm{Cos}\ B$$

$$R^2 = [40^2 + 60^2 - (2 \times 40 \times 60 \times \mathrm{Cos}\ 155^0)]$$

$$\underline{R = 97.73N}$$

(ii)<u>Law Of Sine :</u>

$$\frac{SinA}{Q} = \frac{\sin B}{R}$$

$$\frac{SinA}{60} = \frac{\sin 155°}{97.73}$$

$$\underline{A = 15.04°}$$

$$\propto = 20° + A$$

$$\propto = (20° + 15.04°)$$

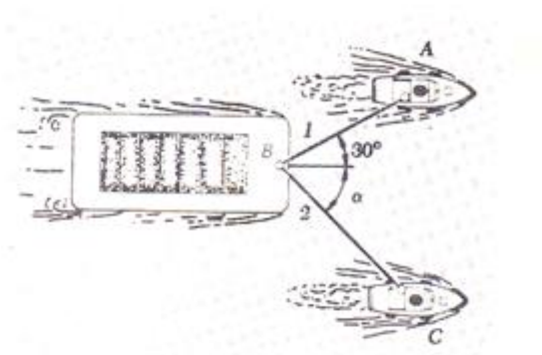
$$\propto = 35.04°$$

<u>Result:</u>

R = 97.73N

$\propto = 35.04°$

1.(b). A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 25 kN force directed along the axis of the barge, determine (i)the tension in each of the ropes, knowing that α=45^0, (ii) the value of α such that the tension in rope 2 is maximum.(8)(May/June 2012)



Given:

Force exerted tug boats = 25KN

$\propto = 45°$

To Find:

(i) T_1 x T_2

(ii) $\propto$ White T_2 is minimum

Solution:

Using Sine Rule,

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{25}{\sin 105°}$$

$$\frac{T_1}{\sin 45°} = \frac{25}{\sin 105°} \Rightarrow T_1 = 18.30KN$$

$$\frac{T_2}{\sin 30°} = \frac{25}{\sin 105°} \Rightarrow T_2 = 12.94KN$$

(ii) The minimum Value of T_2 Occurs When $T_1 \perp T_2$

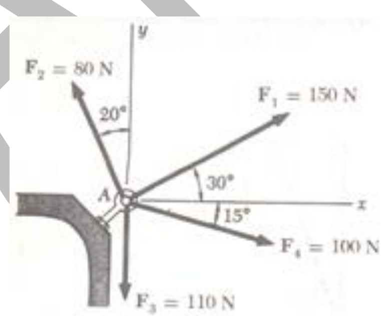
$T_2 = 25 \text{ x Sin } 30^0 \Rightarrow T_2 = 12.24KN$

Corresponding value of T_1 & $\propto$ are,

$T_1 = 25 \text{ x Cos } 30^0 \Rightarrow T_1 = 21.7KN$

$\propto = (90° - 30°) \Rightarrow \propto = 60°$

2.Four forces act on bolt A as shown in figure. Determine the resultant of forces on the bolt. (16)



Given:

F_1 = 150N ; F_2 = 80N ; F_3 = 110N ;

To Find:

Resultant

Force	Magnitude (N)	X-Component	Y-Component
F_1	150	+129.9	75

F_2	80	-27.4	70.2
F_3	110	0	-110.0
F_4	100	96.6	-25.9
		R_X=199.1	R_Y=14.3

R = (R_X.i + R_Y.j)

R = (199.1i + 14.3j)

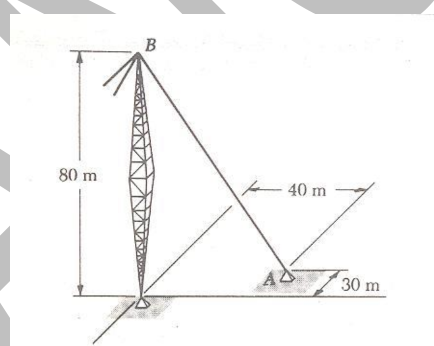
$$\text{Tan} \propto = \frac{R_y}{R_x} = \frac{14.3}{199.1}$$

$$\propto = 4.1^\circ$$

$$R = \frac{14.3}{\sin \propto}$$

$$R = \frac{14.3}{\sin 41^\circ} \Rightarrow R = 199.6N$$

3.A tower guy wire is anchored by means of bolt at A. The tension in the wire is 2500N. Determine (a) the components Fx, Fy,Fz of the force acting in the bolt, (b) the angles θx,θy, θz defining the direction of force. (16)



Given :

F = 2500N ; d_x = -40m ; d_y = 80m ; d_z=30m

To Find:

(a) F_x ,F_y&F_z

(b) $\theta_x, \theta_y \,\&\, \theta_z$

Solution ;

Position of A = (40,0,-30)

B = (0,80,0)

$$\overrightarrow{AB} = (0-40)i + (80-0)\,j + (0-30)\,\text{k}$$

$$\overrightarrow{AB} = -40i + 80\,j + 30k$$

$$AB = \sqrt{-40^2 + 80^2 + 30^2}$$

AB = 94.3m

<u>Total Distance from A to B</u>

AB = $d = \sqrt{dx^2 + dy^2 + dz^2}$

$$AB = \sqrt{-40^2 + 80^2 + 30^2}$$

$$AB = 94.3m$$

<u>Unit Vectors along Co-ordinate axis,</u>

$$\overrightarrow{AB} = -40i + 80j + 20k$$

$$\vec{F} = Fx = F.\ \frac{\overrightarrow{AB}}{AB} = \frac{2500}{94.3}$$

$$\vec{F} = \frac{2500}{94.3}\,[-40i + 80j + 30k]$$

$\vec{F}$ = -1060i + 2120j + 795k

F_x = -1060N

F_y = 2120N

F_x = 795N

<u>(b) Direction of Force:</u>

$$Cos\theta_x = \frac{F_x}{F} = \frac{-1060}{2500}$$

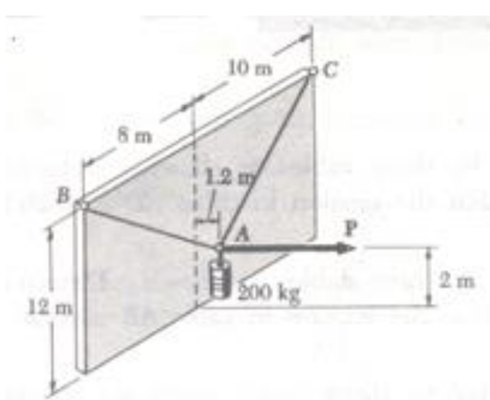
$$\theta_x = 115.1°$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{2120}{2500}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{795}{2500}$$

$$\theta_z = 71.5°$$

4.A 200 Kg cylinder is hung by means of two cables AB and AC, which are attached to the top of the vertical wall. A horizontal force P perpendicular to the wall holds the cylinder in the position shown in figure. Determine the magnitude of P and the tension in each cable. (16) (Nov/Dec 2009)



Given:

A = (1.2, 0, 0)

B = (0, 10, 8)

C = (0, 10,-10)

To Find,

(i) Magnitude of P

(ii) Tension T_{AB} & T_{AC}

Solution:

P = Pi

W = - mgj

W = -1962j

$$\lambda_{AC} = -0.0846i + 0.705j - 0.705k$$
$$T_{AC} = T_{AC}.\lambda_{AC}$$
$$T_{AC} = -0.0846T_{AC}\,j + 0.705T_{AC}\,j - 0.705T_{AC}.k$$

Equilibrium Condition,

$$\sum F = 0$$
$$(T_{AB} + T_{AC} + P + W) = 0$$
$$(-0.0933iT_{AB} - 0.0846T_{AC} + P)i + (0.778T_{AB} + 0.705T_{AC} - 1962)\,j + (0.622T_{AB} - 0.705T_{AC})\,\mathrm{k} = 0$$
$$\sum F_X = 0;$$

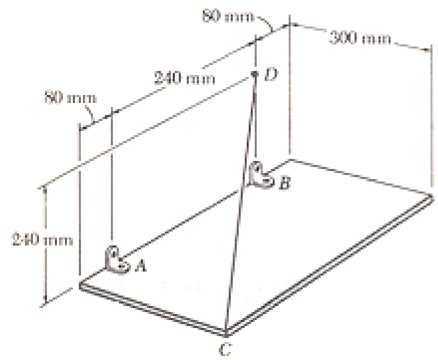
$$(-0.0933T_{AB} - 0.0846T_{AC} + P) = 0$$
$$(0.778T_{AB} + 0.705T_{AC} - 1962) = 0$$
$$(0.622T_{AB} - 0.705T_{AC}) = 0$$

Result:

P=235N

T=140N & 1236N

5.A rectangular plate is supported by bracket at A and B and by a wire CD. Knowing that the tension in the wire is 200N, determine the moment about A of the force exerted by the wire on point C. (16)(Nov/Dec 2009)



Given:

A = (0, 0,-0.08)

B = (0.3, 0, 0)

D = (0, 0.24,-0.32)

To Find:

Moment about A (M_A)

Solution:

$M_A = \frac{\Upsilon_C}{F}$ x F

Where, $\frac{\Upsilon_C}{F}$ is the Vector drawn from A to C

$$\frac{\Upsilon_C}{F} = \overrightarrow{AC} = 0.3i + 0.08k$$

Unit Vector,

$$\lambda = \frac{\overrightarrow{CD}}{CD}$$

F = F λ = 200 $\frac{\overrightarrow{CD}}{CD}$

Resolving the Vector $\overrightarrow{CD}$ into rectangular Component,

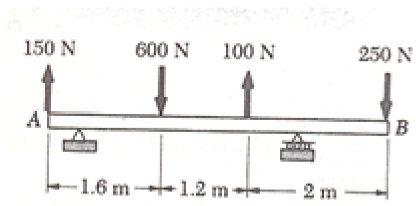
$$\overrightarrow{CD} = -0.3i + 0.24j - 0.32k$$
$$CD = 0.5m$$
$$F = \frac{200}{0.5}\left[-03i + 0.24j - 0.32k\right]$$
$$F = -120i + 96j - 128k$$

Substitute Values,

$M_A = \gamma_C \times F$

$M_A = (0.3i + 0.08k) X(-120i + 96j - 128k)$
$M_A = -7.68i + 28.8j + 28.8k$

6. A 4.8m beam is subjected to the force shown in figure. Reduce the given system of force to (a) an equivalent force couple system at A, (b) equivalent force couple system at B, (c) a single force or resultant.(16)



Solution:

a) Force Couple System at A

$R = \sum F$
$R = (150i - 600j + 100j - 250j)$
$R = -600j \downarrow$

$M_A = \sum(\gamma \times F)$

$M_A = [(1.6i \times (-600j)) + (2.8i \times 100j) + (48i \times (-250j))]$

$M_A = -1880 N.m$

The Equivalent force Couple System at "A"is

$R = 600N \downarrow$
$M_A = 1880 N.m$

(b) force Couple System at "B"

The Force R is Unchanged

$M_B = M_B + (\overrightarrow{BA} \times R)$

$M_B = -1880R + (-4.8i - 600j)$

$M_B = 1000 N.m$

The equivalent force Couple system at "B"

$R = 600N \downarrow$
$M_A = 1000 N.m$

(c) Single Force (or) Resultant:

$$\gamma \text{ x R} = M_A$$

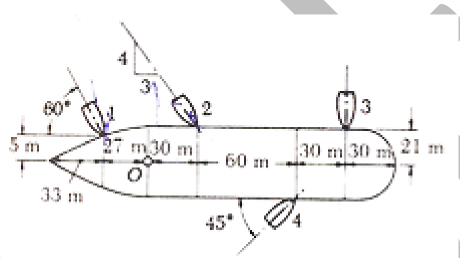
$$xi.(-600j) = -1880$$
$$-x.600 = -1880$$
$$x = 3.13m$$

Thus the single force equivalent to the given system is,

R = 600N↓

X = 3.13m

7. Four tugboats are used to bring an ocean liner to its pier. Each tugboats exerts 30 kN force in the direction shown in figure. Determine (a) the equivalent force couple system at the foremast O, (b) the point on the hull where single, more powerful tugboats should push to produce the same effect as the original four tugboats. (16) (May/ June 2010)



Solution:

(a) Force Couple System at '0'

Resultant, (R) = $\sum F$

$$\sum F = (15i - 26j) + (18i - 24j) + 80j + (2\Phi.2i + 21.2j)$$

$$\sum F = (54.2i - 58.8j)$$

$$M_O = \sum(\gamma \text{ x F})$$

$$M_O = [(-27i + 15j) + (15i - 26j) + (30i + 21j)\text{ x}(12i - 24j) + (120i + 21j)\text{ x}(-30\text{ j}) + (90\text{ j} - 21\text{ j})\text{ x}(21.2i + 21.2j)]$$
$$M_O = (702 - 225 - 720 - 373 - 3600 + 1908 + 445)$$

$$M_O = -1868KN.m$$

R = 80KN

(b) Single tug Boat

$$\gamma = (xi + 21j)$$

(γ x R) = M_o

$$(xi+21j)\,x(54.2i-58.8j)=-186.8k$$
$$-x(58.8)\,R-1138\,k=-1868\,k$$
$$x=12.41\,m$$

8.Four forces of magnitude 10kN, 15kN, 20kN and 40kN are acting at a point O as shown in figure. The angle made by 10kN, 15kN, 20kN and 40kN with X-axis are 30^0, 60^0, 90^0 and 120^0 respectively. Find the magnitude and direction of the resultant force. (16) (Five times)

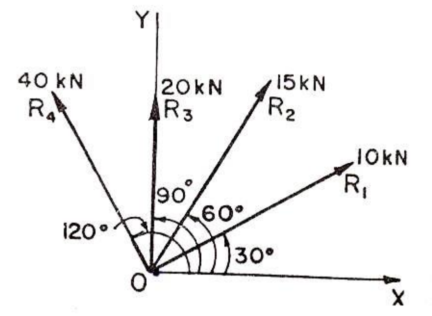


Fig. 1.18

Given:

$$R_1=10KN;\theta_1=30°$$
$$R_2=15KN;\theta_2=60°$$
$$R_3=20KN;\theta_3=90°$$
$$R_4=40KN;\theta_4=120°$$

To find:

Resultant (R) and θ.

Solution:

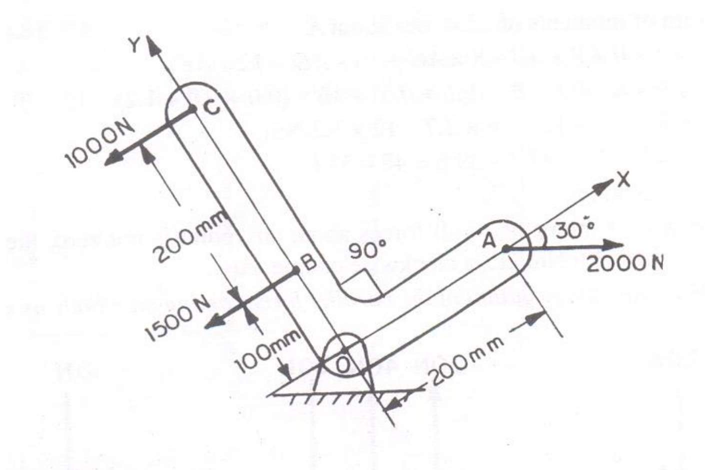
X-axis,

$$H=(R_1\cos\theta_1+R_2\cos\theta_2+R_3\cos\theta_3+R_4\cos\theta_4)$$
$$H=(10\cos 30°+15\cos 60°+20\cos 90°+40\cos 120°)$$
$$H=-8.84KN$$

Along Y-axis,
$$V=R_1\sin\theta_1+R_2\sin\theta_2+R_3\sin\theta_3+R_4\sin\theta_4$$
$$V=10\sin 30°+15\sin 60°+20\sin 90°+40\sin 120°$$
$$V=72.63KN$$
Magnitude of Resultant Force,
$$R=\sqrt{H^2+V^2}$$
$$R=\sqrt{-8.84^2+72.63^2}$$
$$R=72.73KN$$
Direction of force (R)

$$\theta = \frac{V}{H}$$
$$\theta = \frac{72.73}{-3.84}$$
$$\theta = 18.91°$$

9.Three external forces are acting on a L- shaped body as shown in figure. Determine the equivalent system through point O. (16)



<u>Given:</u>

Force at A=2000N

B=1500N

C=1000N

Distance OA=200mm,OB=100mm & BC=200mm.

<u>To Find:</u>

Single resultant (R)

Single Moment (M)

<u>Solution:</u>

The force A is resolved into TWO,

Along X-axis = (2000 X cos30^0) = 1732N

Along Y-axis = (2000 X sin30^0) = 1000N

<u>Resolving All Forces along X-axis,</u>

$$\sum F_X = (2000\cos 30° - 1500 - 1000)$$
$$\sum F_X = -768N$$
$$\sum F_Y = (-2000X\sin 30°)$$
$$\sum F_Y = -1000N$$

Resultant,

$$R = \sqrt{\sum {F_X}^2 + \sum {F_Y}^2}$$
$$R = \sqrt{(-786)^2 + (-1000)^2}$$
$$R = 1260.88N$$

<u>Moment of all forces about point ‘0’ is</u>

$M_o = [(-2000 \sin 30°) \text{X} (200 + 1500) \text{X} (100) + (1000 \text{X} 300)]$

$M_o = 250 N.m$

R = 1260.88N

M_o=250N.m

10.Four forces 32kN, 24kN, 24kN and 120kN are concurrent at origin and are respectively directed through the points whose coordinates are A (2,1,6), B(4,-2,5), C(-3,-2,1), and D(5,1,-2). Determine the resultant of the system. (16) (May/June 2005)

Sol:

Let 'O' be the origin let $\overrightarrow{F_1}, \overrightarrow{F_2}, \overrightarrow{F_3}, \& \overrightarrow{F_4}$ be the Forces along $OA, OB, OC \& OD$ Respectively,

Force along OA $(\overrightarrow{F_1})$ =(magnitude of $\overrightarrow{F_1}$) x (unit vector along OA)

$$= 38 \text{ x} \frac{2iXjX\,6k}{\sqrt{x^2 + 1^2 + 6^2}}$$

$$= 10\vec{i} + 5\vec{j} + 30\vec{k}$$

Similarly, Force along OB $(\overrightarrow{F_2})$

$$= 24X \frac{(4i - 2j + 5k)}{\sqrt{4^2 + (-2)^2 + 5^2}}$$

$$= 14.31\vec{i} - 7.156\vec{j} + 17.89\vec{k}$$

Force, along OC $(\overrightarrow{F_3}) = 24 \text{ x} \dfrac{(-3i - 2j + k)}{\sqrt{(-3)^2 + (-2)^2 + 4^2}}$

$$= -19.25\vec{i} - 12.83\vec{j} + 6.417\vec{k}$$

Force,along OD $(\overrightarrow{F_4}) = 120 \text{ x} \dfrac{(5i + j - 2k)}{\sqrt{5^2 + 1^2 + (-2)^2}}$

$$= 109.55\vec{i} + 21.91\vec{j} - 43.82\vec{k}$$

Resultant of the forces

$$\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \overrightarrow{F_4}$$

$$\vec{R} = (114.61\vec{i} + 6.924\vec{j} + 10.487\vec{k})$$

Magnitude of the resultant $= \sqrt{(114.61^2 + 6.924^2 + 10.487^2)}$

= 115.297 KN

$$\theta_X = \cos^{-1}(\frac{119.61}{115.297})$$

$$\theta_X = 6.28°$$

$$\theta_Y = \cos^{-1}(\frac{6.924}{115.297})$$

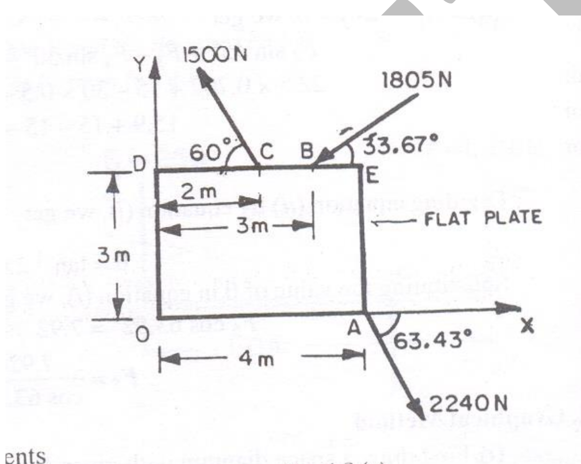
$$\theta_Y = 86.56°$$

$$\theta_Z = \cos^{-1}(\frac{10.487}{115.297})$$

$$\theta_Z = 84.78°$$

Resultant: R = 115.297 K

11.Figure shows the coplanar system of forces acting on a flat plate. Determine (i) the resultant and (ii) x and y intercepts of the resultant. (16) (May/June 2010)



Given:

Force at A=2045N

Angle with X-axis = 63.43^0

Force at B=1805N

Angle with X-axis = 60^0

Lengths, OA=4m,OB=3m,OC=2m & OD=3m

To Find:

(i) Resultant (R)

(ii) X & Y intercepts of resultant,

Solution:

(i) Force at A = 2240 N

X-Component = (2240 X Cos 63.43^0) = 1502.2 N

Y-Component =(2240 X Sin 63.43^0) = 2003.4 N

Force at B = 1805 N

X-Component = (1805 X Cos 33.67^0) = 1502.2 N

Y-Component = (1805 X Sin 33.67^0) = 1000.7 N

Force at C = 1500N

X-Component = (1500 X Cos 60^0) = 750 N

Y-Component = (1500 X Sin 60^0) = 1299 N

Force along X-axis,

$$R_X = \sum F_X = [1001.9 + 1500 + (-750)] = 1250.3N$$

Force along Y-axis,

$$R_Y = \sum F_Y = (-2003.4 - 1000.7 + 1299) = -1705.1N$$

Resultant force is given by,

$$R = \sqrt{R_X{}^2 + R_Y{}^2} = \sqrt{(-1250.3)^2 + (-1705.1)^2}$$

$$R = 2114.4N$$

The along made by the resultant with X-axis is given by ,

$$Tan\theta = \frac{R_Y}{R_X} = \frac{-1705.1}{-1250.3}$$

$$\theta = 53.7°$$

(ii) Intercepts of Resultant along X-axis & Y-axis.

Moment of R about 0 = Sum of moments of R_X = R_Y at 0

$$-2411.1 = (R_X X 0) + (R_Y.x)$$

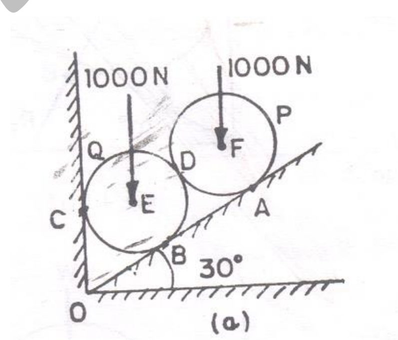
$$X = \frac{-2411.1}{-1705.1} = 1.41m$$

Moment of R about 0 = Sum of moments of R_X = R_Y at 0

$$-2411.1 = (R_X.Y) + (R_Y X 0)$$

$$X = \frac{-2411.1}{-1250.3} = 1.92m$$ Below D.

12.Two identical rollers, each of weight W=1000N, are supported by an inclined plane and a vertical wall as shown in figure. Find the reactions at the points of supports A,B and C. Assume all the surfaces to be smooth. (16) (Nov/Dec 2009)



Given:

Weight of each roller = 1000N
Radius of each roller same

To find:

Reactions at points of supports A,B& C

Solution:

Equilibrium of Roller (P):

The resultant force in X and Y directions on roller "P" should be Zero.

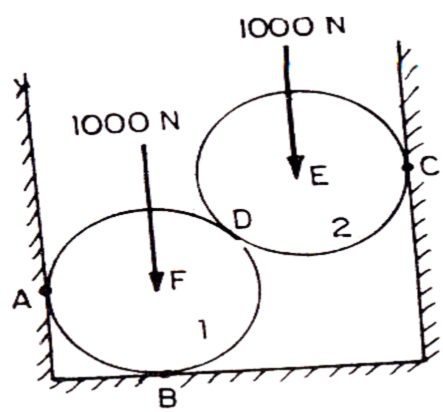
$$\sum F_X = 0$$
$$(R_D.\sin 60° - R_A.\sin 30°) = 0$$
$$R_D = 0.577 R_A$$
$$\sum F_Y = 0$$
$$(R_D.\cos 60° - R_A.\cos 30° - 1000) = 0$$
$$(0.577 R_A.\cos 60° + R_A.\cos 30° - 1000) = 0$$
$$R_A = 866.17 N$$

Equilibrium of Roller (Q):

$$\sum F_X = 0$$
$$(R_B.\sin 30° - R_B.\sin 60° - R_C) = 0$$
$$(R_B X 0.5 + 499.78.\sin 60° - R_C)$$
$$R_C = (0.5 R_B + 499.78)$$
$$\sum F_Y = 0$$
$$(R_B.\cos 30° - 1000 - R_D.\cos 60°) = 0$$
$$(R_B X \cos 30° - 1000 - 499.78.\cos 60°) = 0$$
$$R_B = 1443.3 N$$

R_C = 1154.45 N

13.Two spheres, each of weight 1000N and of radius 25cm rest in a horizontal channel of width 90cm as shown in figure. Find the reactions on the points of contact A,B and C. (16) (Nov/Dec 2012)



Given:

Weight of Each sphere, W = 1000 N

Radius, r = 25 cm

Width of channel = 90 cm

To Find:

Reaction on the points of contact A,B and C.

Solution:

AF = BF = FD = CE = 25 cm

EF = (25 + 25) = 50cm ; FG = 40 cm

In Δ^{le}, EFG,EG = $\sqrt{EF^2 - FG^2} = \sqrt{50^2 - 40^2}$ = 30 cm

Equilibrium of sphere (2):

$$\sum F_X = 0$$
$$R_D.\sin\theta = R_C$$
$$\sum F_Y = 0$$
$$R_D.\cos\theta = 1000$$
$$R_D = (\frac{1000}{\cos\theta})$$
$$R_D = (\frac{1000}{\frac{3}{5}}) = \frac{5000}{3}$$
$$R_C = (\frac{5000}{3} X \frac{4}{5})$$
$$R_C = 1333.33N$$

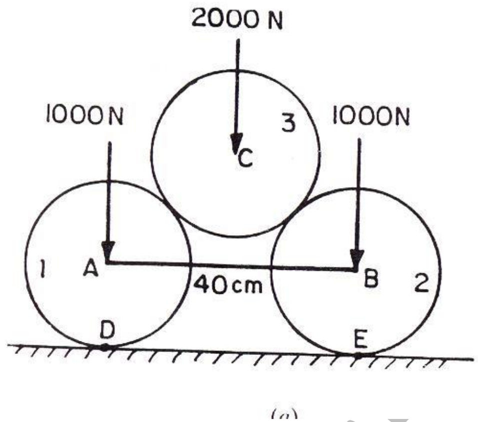
Equilibrium of sphere (1):

$$\sum F_X = 0$$
$$(R_A - R_D.\sin\theta) = 0$$
$$R_A = (\frac{5000}{3} X \frac{4}{5})$$
$$R_A = 1333.33N$$

$$\sum F_Y = 0$$
$$(R_B - 1000 - R_D.\cos\theta) = 0$$
$$R_B = (1000 + \frac{5000}{3} X \frac{3}{5})$$
$$R_B = 2000N$$

14.Two smooth circular cylinders, each of weight W=1000N and radius 15 cm, are connected at their centres by a string AB of length=40 cm and rest upon a horizontal plane,

supporting above them a third cylinder of weight =2000N and radius 15cm as shown in figure. Find the force S in the string AB and the pressure produced on the floor at the points of contact D and E. (16)



Solution:

$$AC = (\text{AH} + \text{FC})$$

$$AC = (15+15) = 30cm$$

$$AH = (\frac{1}{2}.AB)$$

$$AH = (\frac{1}{2} X\,40) = 20cm$$

From Δ^{le} ACH ,

$$\sin\theta = \frac{AH}{AC} = \frac{20}{30} = 0.667$$

$$\theta = \sin^{-1}(0.667) \Rightarrow \theta = 41.836°$$

Equilibrium of cylinder (3):

Resolving forces horizontally ,

$$(R_F.\sin\theta - R_G.\sin\theta) = 0$$

$$R_F = R_G$$

Resolving forces vertically,

$$(R_F.\cos\theta + R_G.\cos\theta) = 2000$$

$$R_F = \frac{2000}{\cos 41.836°}$$

$$R_F = 1342.179N$$

Equilibrium of cylinder (1):

$$\sum F_X = 0$$

$$S - R_F.\sin\theta = 0$$

$$S = (1342.179X \sin 41.836°)$$

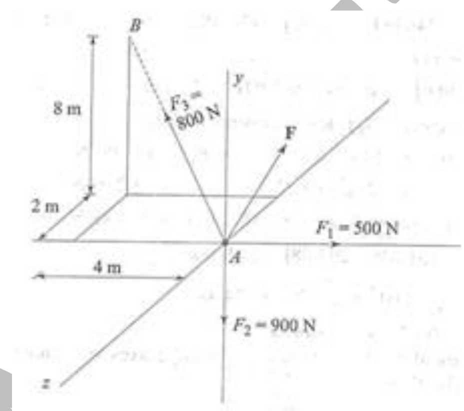
$$S = 895.2N$$

$$\sum F_Y = 0$$
$$(R_D - 1000 - R_F.\cos\theta) = 0$$
$$R_D = 1000 + R_F.\cos\theta$$
$$R_D = (1000 + 1342.179X\cos 41.836°)$$
$$R_D = 2000N$$

Equilibrium of cylinder 1,2& 3 taken together:

$$(R_D + R_E - 1000 - 2000 - 1000) = 0$$
$$R_E = (4000 - R_D)$$
$$R_E = 2000N$$

15.Determine the magnitude and direction of force F shown in figure so that the particle A is in equilibrium. (16)



Solution:

For the particle A to be in Equilibrium $\sum F = 0$,Must be satisfied.

$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F} = 0$$
$$F_1 = 500i; F_2 = -900j; F_3 = F_3\lambda AB,$$

Where,

$$\lambda AB = \frac{\overrightarrow{AB}}{AB}$$
$$\lambda AB = \frac{-4i + 8j - 2k}{\sqrt{(-4)^2 + 8^2 + (-2)^2}}$$
$$\lambda AB = \frac{1}{9.17}(-4i + 8j - 2k)$$
$$F_3 = \frac{800}{9.17}(-4i + 8j - 2k)$$
$$F_3 = -345.96i + 697.92j - 174.48k$$
$$\vec{F} = F_X i + F_Y j + F_Z K$$

=(500i - 900j - 348.96i + 697.92j - 174.48k + $F_X i + F_Y j + F_Z K$)=0

Equating the respective i, j, k components to zero,

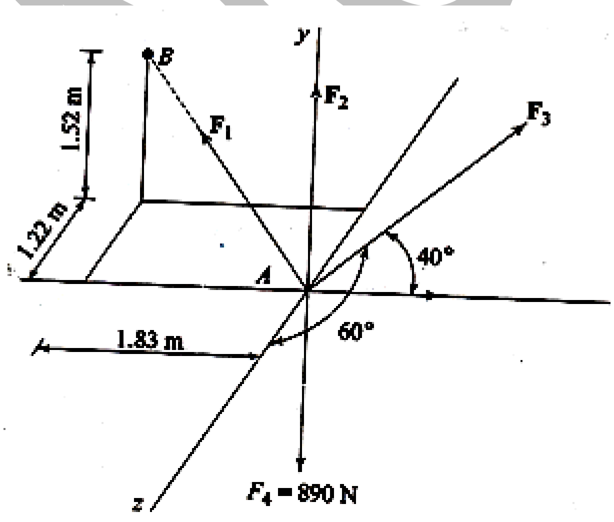
$$\sum F_X = 0 \Rightarrow 151.04 + F_X = 0 \Rightarrow F_X = -151.04N$$
$$\sum F_Y = 0 \Rightarrow -202.08 + F_Y = 0 \Rightarrow F_Y = 202.08N$$
$$\sum F_Z = 0 \Rightarrow -174.48 + F_Z = 0 \Rightarrow F_Z = 174.48N$$
$$F = (-151.04i + 202.08j + 174.48k)$$
$$F = \sqrt{(151.04)^2 + (202.08)^2 + (174.48)^2}$$
$$F = 306.75N$$

Direction of force (F):

$$\cos\theta_X = \frac{F_X}{F} = \frac{-151.04}{306.75}$$
$$\theta_X = 119.5°$$
$$\cos\theta_Y = \frac{F_Y}{F} = \frac{202.05}{306.75}$$
$$\theta_Y = 48.8°$$
$$\cos\theta_Z = \frac{F_Z}{F} = \frac{174.48}{308.75}$$
$$\theta_Z = 55.3°$$

16. Determine the magnitudes of forces F1,F2 and F3 for equilibrium of the particle A shown in figure.

(16)



Solution:

A (0,0,0)
B (-1.83,1.52,-1.22)
$F_1+F_2+F_3+F_4=0$

F1 = F1 $\lambda_{AB} = \frac{\overrightarrow{AB}}{AB}$

$$\overrightarrow{AB} = (xi + yj + zk)$$
$$\overrightarrow{AB} = -1.83i + 1.52j - 1.22k$$
$$AB = \sqrt{(-1.83)^2 + (1.52)^2 + (-1.22)^2}$$
$$AB = 2.67m$$
$$\lambda AB = [-0.69i + 0.57j - 0.46k]$$
$$F_1 = F_1[-0.69i + 0.57j - 0.46k]$$
$$F_2 = F_2 j$$
$$F_3 = (F_3 \cos 40° i + F_3 \cos 50° j + F_3 \cos 60° k)$$
$$F_3 = F_3(0.77i + 0.64j + 0.5k)$$
$$F_4 = -890j$$

$$[F_1(-0.69i + 0.57j - 0.46k) + F_2 j + F_3(0.77i + 0.64j + 0.5k) - 890j] = 0$$

Equating the respective i,j& k components to zero,

$-0.69F_1 + 0.77F_3 = 0$

$0.57F_1 + F_2 + 0.64F_3 = 890$

$-0.46F_1 + 0.50F_3 = 0$

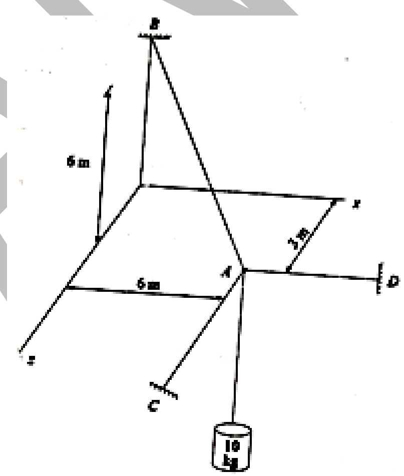
Solving above equations,

$F_1 = 0$

$F_3 = 0$

$F_2 = 890N$

17.Three cables are used to support the 10 kg cylinder shown in figure. Determine the force developed in each cable for equilibrium. (16) (Nov/Dec 2009)



Solution,

A (6 , 0 , 3) ; B (0 , 6 , 0)

Let us choose point A as & free body,

Let , the tension in the cables be F_1, F_3& F_4 as shown in figure.

For equilibrium,

$F_1+F_2+F_3+F_4=0$

$F_1 = F_1 i$; $F_2 = 98.1j$; $F_3 = F_3 k$;$F_4=F_4 \lambda AB$

$$\lambda AB = \frac{\overrightarrow{AB}}{AB}$$
$$AB = xi + yj + zk$$
$$\overrightarrow{AB} = -6i + 6j - 3k$$
$$AB = \sqrt{(-6)^2 + 6^2 + (-3)^2}$$
$$AB = 9m$$
$$\lambda AB = (-0.67i + 0.67j - 0.33k)$$
$$F_4 = (-0.6i + 0.67j - 0.33k)F_4$$

$$[F_1 i - 98.1j + F_3 k + (0.67i + 0.67j - 0.33k)F_4] = 0$$

Equitation the respective I,j,k Components to Zero,

$(F_1 - 0.67F_4) = 0$

$(-98.1 + 0.67\ F_4) = 0$

$(F3 - 0.33F_4) = 0$

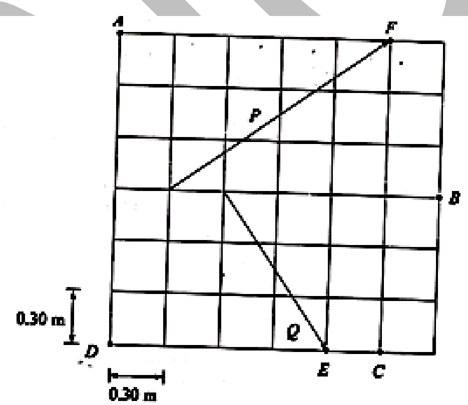
Solving above equations

$F_1 = 98.1N$

$F_2 = 48.32N$

$F_3 = 146.42N$

18. Compute the moment of the force P=2000N and of the force Q=1600N shown in figure about points A,B,C and D. (16)



Solution:

The inclination of force (P) about the horizontal axis is θ_1

$$\theta_1 = Tan^{-1}(\frac{0.9}{1.2})$$
$$\theta_1 = 36.9°$$

The inclination of force (Q) about the horizontal axis is θ_2

$$\theta_2 = Tan^{-1}(\frac{0.9}{0.6})$$
$$\theta_2 = 56.3°$$

Moment about "A"

Moment of (P) about A,

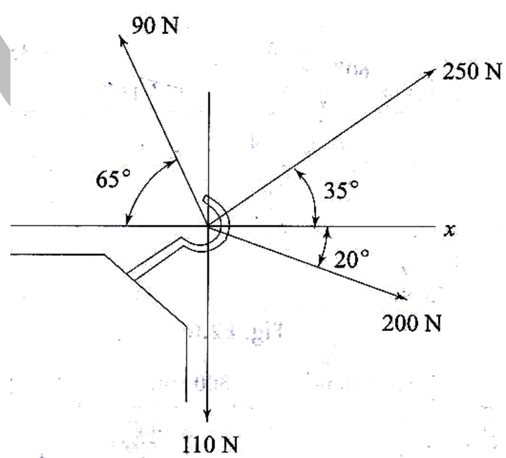
$$M_A = [1.5iX(160i + 1200j)]$$
$$M_A = 1800 N.\text{m}$$
Moment about "B"

Moment of force (P) about B,
$$(M_B)_P = (-0.3i + 0.9j)\,\text{x}(1000\text{i} + 1200j)$$
$$(M_B)_P = (-360 - 1440) = -1800 N.m$$
$$(M_B)_P = 1800 N.m$$
Moment of force (Q) about "B"
$$(M_B)_Q = [(-0.6i - 0.9j)X(888i - 1331j)]$$
$$(M_B)_Q = (798 + 798) = 1598 N.m$$
$$(M_B)_Q = 1598 N.m$$
Moment about (C):

Moment of force (P) about C,
$$(M_C)_P = [1.8jX(1600i + 1800j)] = -2880 N.m$$
$$(M_C)_P = 2880 N.m$$

Moment of force (Q) about "C"
$$(M_C)_Q = [(-0.3i)X(888i - 1331j)] = 399 N.m$$
$$(M_C)_Q = 399 N.m$$
Moment of force (P) about D,
$$(M_D)_P = [(1.5i + 1.8j)X(1600i + 1200j)]$$
$$(M_D)_P = (1800 - 2880) = -1080 N.m$$
Moment of force (Q) about "D"
$$(M_D)_Q = [1.2iX(888i - 1331j)] = 1597 N.m$$
$$(M_D)_Q = 1597 N.m$$

19. Determine the magnitude and direction of the resultant of the forces acting on the hook shown in figure. (16)

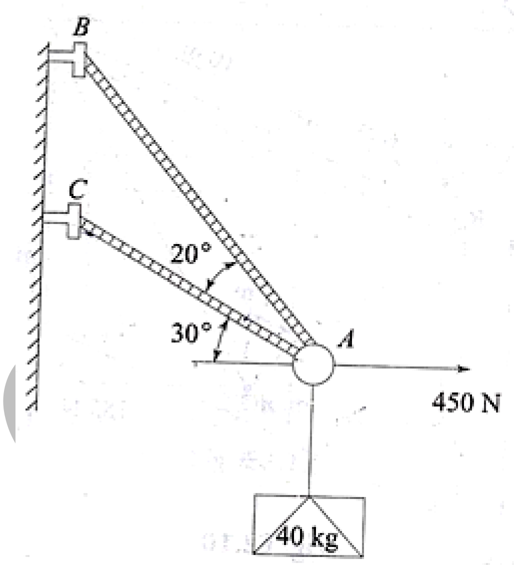


Solution:

Applying equilibrium condition.

$$\sum F_X = 0$$
$$(200\cos 30° + 250\cos 35° - 90\cos 65°) = 354.69$$
$$\sum F_Y = 0$$
$$(250\sin 35° + 90\sin 65° - 200\sin 30 - 100) = 56.55$$
$$R = \sqrt{\sum F_Y{}^2 + \sum F_X{}^2}$$
$$R = \sqrt{56.55^2 + 354.69^2}$$
$$R = 359.17N$$
$$Tan\theta = \frac{\sum F_Y}{\sum F_X}$$
$$Tan\theta = \frac{56.55}{359.17}$$
$$\theta = Tan^{-1}(\frac{56.55}{359.17})$$
$$\theta = 8°$$

20. Determine the tension in cables AB and AC required to hold the 40kg crate show in figure. (16) (May/June 2004)



Solution:

Applying equilibrium Condition,

$$\sum F_X = 0$$
$$(450 - T_c.\cos 30° - T_B\cos 35°) = 0$$
$$(\mathrm{T}_C\cos 30° + T_B\cos 50°) = 450$$

$$\sum F_Y = 0$$

$$(T_c \sin 30° + T_B \sin 50°) = 392.4$$

$$T_c = \frac{392.4 - T_B \sin 50°}{\sin 30°}$$

From equations (1) & (2)

450 = $\left(\frac{392.4 - T_B \sin 50°}{\sin 30°}\right)$ COS 30^0 + T_B COS 50^0

T_B = 333.79N

From Equations (1),

T_C COS 30^0 + (333.79 X COS 50^0) = 450

T_C = 270.72N

Unit 2

FREE BODY DIAGRAM

- A **free body diagram**, sometimes called a force diagram, is a pictorial device, often a rough working sketch, used by engineers and physicists to analyze the forces and moments acting on a body.
- Drawing a free body diagram can help determine the unknown forces on, moments applied to, and equations of motion of, the body and thus help to analyze a problem in statics or dynamics.
- Draw a simplified version of the object (does not have to be a square or a circle, it can be anything).
- Present force vectors with arrows. (Sometimes length of the arrow is proportional to the force)
- Draw the arrows from the center of the object.
- Label forces.
- Body (or any part of it) which is currently stationary will remain stationary if the resultant force and resultant moment are zero for all the forces and couples applied on it.
- **Newton's 3rd law:** Each action has a reaction equal in magnitude and opposite in direction. This law provides the method used for one body (or part of a body) to interact with another body (or another part of the body).
- Select the body (or part of a body) that you want to analyze, and draw it.
- Identify all the forces and couples that are applied onto the body and draw them on the body. Place each force and couple at the point that it is applied.
- Label all the forces and couples with unique labels for use during the solution process.
- Add any relevant dimensions onto your picture.

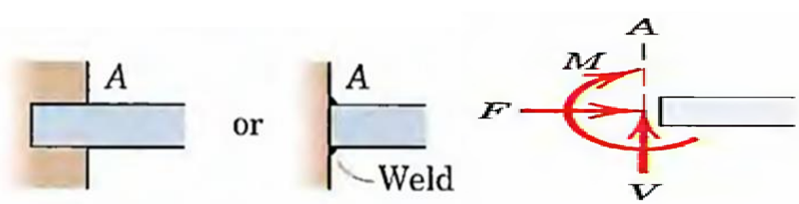
- **Varignon's theorem** is a statement in Euclidean geometry by Pierre Varignon that was first published in 1731. It deals with the construction of a particular parallelogram (**Varignon parallelogram**) from an arbitrary quadrangle (quadrilateral).
- The midpoints of the sides of an arbitrary quadrangle form a parallelogram. If the quadrangle is convex or reentrant, i.e. not a crossing quadrangle, then the area of the parallelogram is half the area of the quadrangle.
- The Varignon parallelogram is a rhombus if and only if the two diagonals of the quadrilateral have equal length, that is, if the quadrilateral is an equi diagonal quadrilateral.

The Varignon parallelogram is a rectangle if and only if the diagonals of the quadrilateral are perpendicular, that is, if the quadrilateral is an orthodiagonal quadrilateral

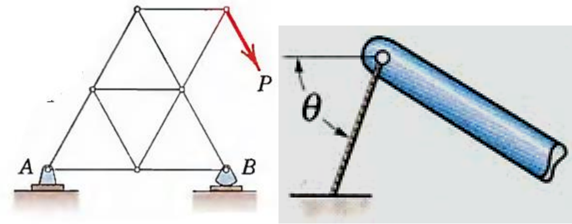
1. Write short notes on free body diagram with one example.

Free body diagram is the isolated diagram of an objectofobjectspoint in the system in which all forces at couple moment acting on it are shown including support reactions.

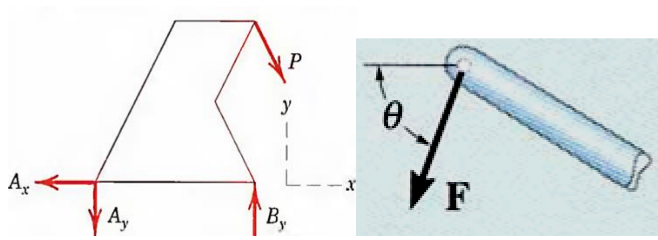
Example:consider a roller support.



1. Draw the free body diagram for the following.



Free body diagram:

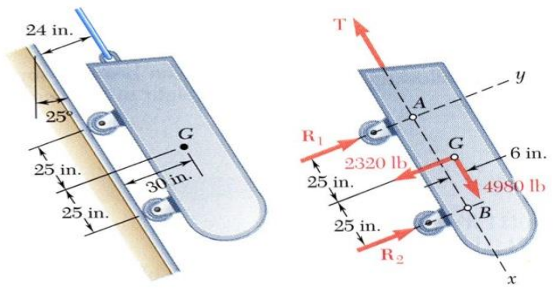


2. Draw the roller support diagrams. (Apr 2010)

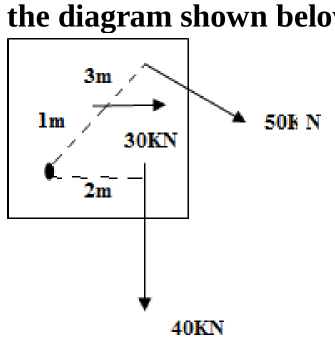
Roller supports:

3. **Draw free body diagram of the given picture**.

Free body diagram:



4. Find the moment about A in the diagram shown below.



<u>Solution</u>:

Moment about A is M=rXF
=(30X1)+503+(40X2)

=260NM

5. List out the types of supports. (may/jun 2006)

There are three types of supports such as

a) Hinge support. (Or) pin support.

b) Roller support.

c) Fixed support.

d) Double overhanging

e) Continuous

f) Trussed

6. List out the types of beams. (apr/may 2011)

Types of beams:

a) Cantileverbeam.

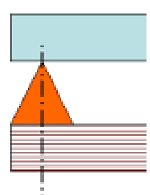
b) Fixed beam.

c) Simply supported beam and

d) Over hanging beam

7. Write short notes on the hinged support and give example.

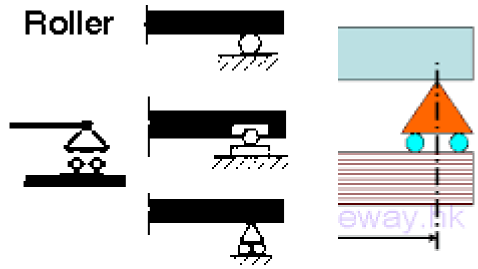
A pinned support can resist both vertical and horizontal forces but not a moment. They will allow the structural member to rotate, but not to translate in any direction. Many connections are assumed to be pinned connections even though they might resist a small amount of moment in reality. It is also true that a pinned connection could allow rotation in only one direction; providing resistance to rotation in any other direction. The knee can be idealized as a connection which allows rotation in only one direction and provides resistance to lateral movement. The design of a pinned connection is a good example of the idealization of the reality. A single pinned connection is usually not sufficient to make a structure stable. Another support must be provided at some point to prevent rotation of the structure. The representation of a pinned support includes both horizontal and vertical forces.



Hinged support.

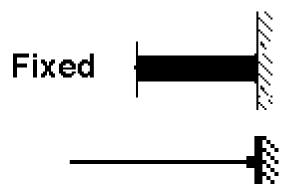
8. **Write short notes the roller support and give example**.

Roller supports are free to rotate and translate along the surface upon which the roller rests. The surface can be horizontal, vertical, or sloped at any angle. The resulting reaction force is always a single force that is perpendicular to, and away from, the surface. Roller supports are commonly located at one end of long bridges. This allows the bridge structure to expand and contract with temperature changes.



9. Write short notes the fixed support and give example.

Fixed connections are very common. Steel structures of many sizes are composed of elements which are welded together. A cast-in-place concrete structure is automatically monolithic and it becomes a series of rigid connections with the proper placement of the reinforcing steel. Fixed connections demand greater attention during construction and are often the source of building failures.

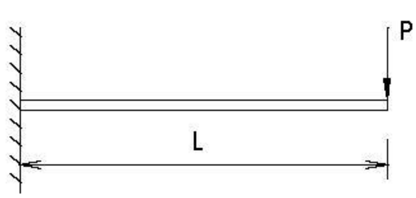


10. Write short notes on beam.

A beam is a structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment. Beams are traditionally descriptions of building or civil engineering structural elements, but smaller structures such as truck or automobile frames, machine frames, and other mechanical or structural systems contain beam structures that are designed and analyzed in a similar fashion.

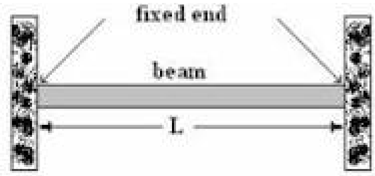
11. Write short notes the cantilever beam and give example.

A cantilever is a beam anchored at only one end. The beam carries the load to the support where it is forced against by a moment and shear stress. Cantilever construction allows for overhanging structures without external bracing. Cantilevers can also be constructed with trusses or slabs. This is in contrast to a simply supported beam such as those found in a post and lintel system. A simply supported beam is supported at both ends with loads applied between the supports



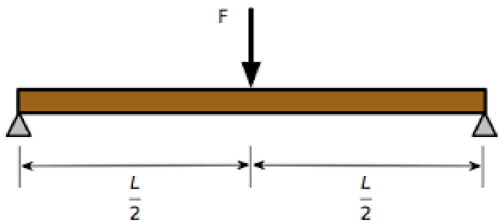
12. Write short notes the fixed beam and give example.

This type of beam is both ends are fixed. Structural beam with fixed, as opposed to hinged, connections.Shown fig.



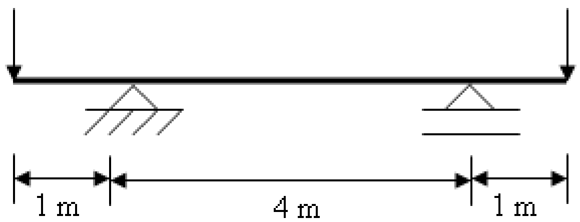
13. Write short notes the simply supported beam and give example.

A simply supported beam is a type of beam that has pinned support at one end and roller support at the other end. Depending on the load applied, it undergoes shearing and bending. It is the one of the simplest structural elements in existence.



14. **Write short notes the Overhanging beam beam and give example**.

A overhanging beam is a beam that has one or both end portions extending beyond its supports. It may have any number of supports. If viewed in a different perspective, it appears as if it is has the features of simply supported beam and cantilever beam.



15. How do you differentiate moment and couples while calculating problems.

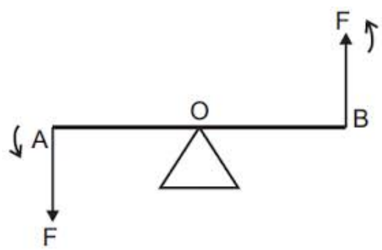
Moment of a force about a point is the measure of its rotational effect. Moment is defined as the product of the magnitude of the force and the perpendicular distance of the point from the line of action of the force from that point.

Two parallel forces equal in magnitude and opposite in direction and separated by a definite distance are said to form a couple. The sum of the forces forming a couple is zero, since they are equal and opposite which means the translatory effect of the couple is zero,

MOMENTS AND COUPLES – MOMENT OF A FORCE ABOUT A POINT AND ABOUT AN AXIS

16. Define couple.

Torque is calculated by the product of either of forces forming the couple and the arm of the couple. i.e.) Torque = one of the force x perpendicular distance between the forces.



17. Why the couple moment is said to be a free vector?

Couple moment is said to be a free vector as it can be transferred to any point in the plane without causing any change in its effect on the body. In mechanics, a couple is a system of forces with a resultant moment but no resultant force. A better term is force couple or pure moment. Its effect is to create rotation without translation, or more generally without any acceleration of the centre of mass. In rigid body mechanics, force couples are free vectors, meaning their effects on a body are independent of the point of application.

18. What is meant by force-couple system?

A system of coplanar non concurrent force system acting in a rigid body can be replaced by a single resultant force and couple moment at a point known as force couple system. Two force systems are equivalent if they result in the same resultant force and the same resultant moment.

19. When is moment of force maximum about a point?

Moment of force is maximum about a point when,

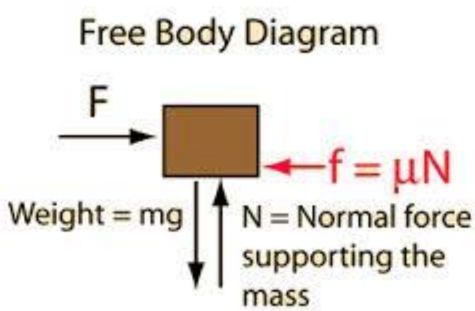
I) It's applied at maximum result from the point and,

ii) It is applied perpendicular to the line joining the point to the point of application of force.

20. Write short notes the free body diagram with one example. (Nov/dev-2012)

A free-body diagram or isolated-body diagram is useful in problems involving equilibrium of forces.

Free-body diagrams are useful for setting up standard mechanics problems.



21. State the necessary and sufficient conditions for equilibrium of rigid bodies in two dimensions.

The necessary and sufficient conditions for equilibrium of rigid bodies in two dimensions are:

1) Algebraic sum of horizontal components of all forces acting on the body is must be zero,

2) Algebraic sum of vertical components all forces acting on the bodies must be zero,

3) Algebraic sum of moments due to all forces and couple moments acting the

22. Write the conditions equilibrium of a system of parallel force acting in a plane.

The two conditions of equilibrium of a system of parallel forces acting in a plane are:

1) Algebraic sum of all forces must be zero,

2) Algebraic sum of moments due to all forces about any point must be zero.

23. Write short notes on Resultant.

Definition: A resultant of number of forces acting on a body is a single. Force which can produce the same effect on the body as it is produced by all the forces acting together. Resultant (net) force causes the displacement of a body (i.e. body moves). The set of forces which causes the displacement of a body are called as component of resultant or component forces.

24. Write short notes on equilibrant.

Definition: An equilibrant of number of forces acting on a body is a single force which cancels the effect of resultant of a system of forces or which brings the system and the body is equilibrium. Equilibrant keeps the body at rest (i.e. in equilibrium). The set of forces which keeps the body at restate known as equilibrium forces or components of equilibrant.

25. State the analytical conditions for equilibrium of coplanar forces in a plane.

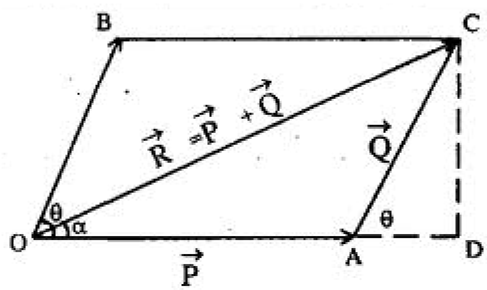
The two conditions for equilibrium of coplanar forces are:.

a. The algebraic sum of all the forces of a force system is equal to zero.$\sum F = 0$

b. The algebraic sum of the moments of all the forces is equal to zero.$\sum M = 0$

26. Write short notes on parallelogram law of forces.

The law of parallelogram of forces states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point

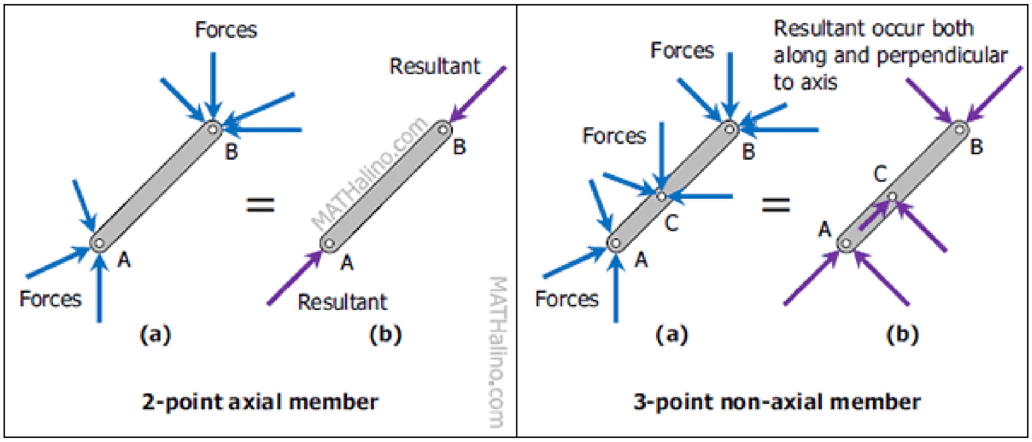


27. **Write short notes on trusses**.

In engineering, a truss is a structure that "consists of two-force members only, where the members are organized so that the assemblage as a whole behaves as a single object “Although this rigorous definition allows the members to have any shape connected in any stable configuration, trusses typically comprise five or more triangular units constructed with straight members whose ends are connected at joints referred to as nodes. In this typical context, external forces and reactions to those forces are considered to act only at the nodes and result in forces in the members which are either tensile or compressive forces. For straight members, moments (torques) are explicitly excluded because, and only because, all the joints in a truss are treated as revolutes, as are necessary for the links to be two-force members.

28. Write short notes the term of frames.

Three-force member is in general a non-axial member that is not simply in tension or compression. A member of this kind has shear forces perpendicular to the member and subjected to bending loads. If forces are applied to more than two positions on the member, it is three-force member. Any beam is a three-force member according to the above definition.



29. Write short notes on equilibrium.

The condition of a system in which all competing influences are balanced, in a wide variety of contexts.

∑Fx=0,

∑Fy=0,

∑M=0.

30. What are the advantages of fixed beam?

A beam that is supported at both ends. It allows neither vertical movement nor rotation at the supports. This is the basic difference between a fixed beam and simply supported beam. So in a fixed beam the supports generate vertical reactions as well as rotational moments.

Either of the beams can be used in practical applications depending on the design criteria and requirement.

31. Write short notes about Moment of a Force about an Axis.

Two forces of the same size and direction acting at different points *are not equivalent.* They may cause the same translation, but they cause different rotation. For straight members, moments (torques) are explicitly excluded because, and only because, all the joints in a truss are treated as revolute, as are necessary for the links to be two-force members.

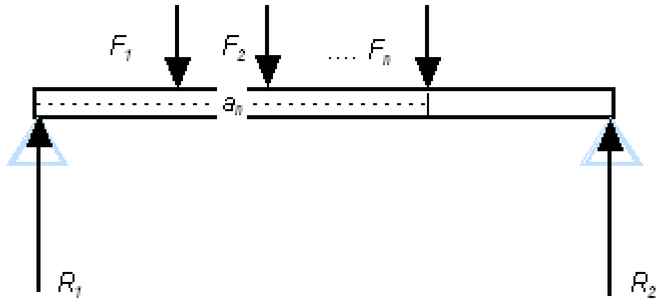
32. What are the types of loads?

There are three types of loads such as,

a) Point load (OR) concentrated load,
b) Uniform distributed load, and
c) Uniform varying load.

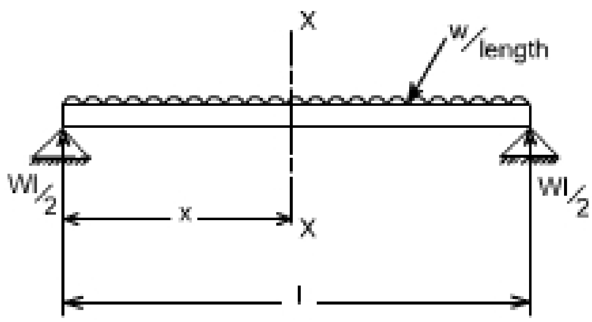
33. Write short notes on point load.

A term used in structural analysis to define a concentrated load on a structural member. Forces like FI, F2, F3….etc.Shown fig.



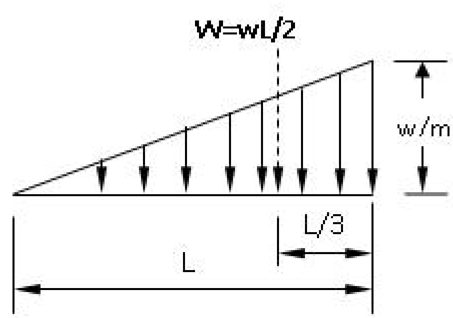
34. Write short notes on uniform distributed load (UDL).

This types of load is ignoring the horizontal forces, a simply supported beam with concentrated load P1 at the middle of the beam and uniformly distributed load of w per unit length over the span L(1-2) of the beam.



35. Write short notes on uniform varying load.

These are the load varying uniformly from zero to a particular value and spread over a certain length of the beam. Such load is also called triangular load. The total load can be obtained by calculating the total area of triangle & multiplied if by the intensity or rate of loading. The total load will act through the centroid of the triangle.



36. What are the symbols are using in equilibrium condition?

Equilibrium condition symbols:

a) Right side force indicating "+ve"
b) Left side force indicating "-ve"
c) Upward force "+ve"
d) Downward force negative "-v"
e) Moment clockwise "-ve"
f) Moment anti clockwise "+ve"

REQUIREMENTS OF STABLE EQUILIBRIUM

37. What is stable equilibrium?

A body is said to be in stable equilibrium, if it returns back to its original position after it is slightly displaced from its position of rest.

38. What is unstable equilibrium?

A body is said to be in unstable equilibrium, if it does not returns back to its original position, and heels farther away after slightly displaced from its position of rest.

39. What is neutral equilibrium?

A body is said to be in neutral equilibrium, if it occupies a new position (also remains at rest) after slightly displaced from its position of rest.

40. What are the characteristics of a couple?

There are two types of characteristics of a couple

i) The algebraic sum of the forces is zero

ii) The algebraic sum of the moments of the forces about any point is the same and equal to the moment of the couple itself.

41. Define moment of a force in beams.

The moment of a force about a point is defined as the turning effect of the force about that point

Moment = force X perpendicular distance

M_A=F X D

42. What condition the moment of a force will be zero? (may/jun-2013)

A force produces zero moment about an axis or reference point which intersects the line of action of the force.

EQUILIBRIUM OF RIGID BODIES IN THREE DIMENSIONS – EXAMPLES

43. What is the difference between a moment and a couple?

Difference between moment and couple:

The couple is a pure turning effect which may be moved anywhere in its own plane, or into parallel plane without change of its effect on the body,

The moment of a force must include a description of the reference axis about which the moment is taken.

44. What are the common types of supports used in three dimensions?

There are three types of supports used in three dimensions such sa:

a) Ball support
b) Ball and socket support,
c) Fixed or welded support and
d) Hinged supports

Vectorial representation of moments and couples – Scalar components of a moment – Varignon's theorem – Equilibrium of Rigid bodies in two dimensions

45. A force (10i+20j-5k)N applied at A (3,0,2)m is moved to point B(6,3,1)m. Find the work done by the force.

Work done= force X displacement

Displacement r = (X2-X1)i +(Y2-Y1)j+(Z2-Z1)

= (6-3)i+(3-0)j+(1-2)k

= 3i+3j-k

$$= \begin{vmatrix} i & j & k \\ 3 & 3 & -1 \\ 10 & 20 & -5 \end{vmatrix} = \underline{\mathbf{5i+5j+30k}} \rightarrow$$

46. A force of magnitude 200N is acting along the line joining P(2,4,6)m and Q(4,7,10)m. find the moment of the force about (7,10,15).

P= (2,4,6) m, X1= 2m, Y1= 4m, Z1=6m

Q = (4,7,10) m X2=4m, Y2= 7m Z2= 10m

FAB =200N

= (X2-X1)i +(Y2-Y1)j+(Z2-Z1)k

=(4-2)i+(7-4)j+(10-6)k

= 2i+3j+4k λ AB = γAB/ IγI = 2i+3j+4k / √ (22+(32)+(42))

= 2i+3j+4k / √ (29) →

FAB = FAB x λ AB

= 100 x (2i+3j+4k / √ (29))

= 74.28i+111.42j+148.56k

Moment Mo = r XF

γAc= (X2-X1)i +(Y2-Y1)j+(Z2-Z1)k.

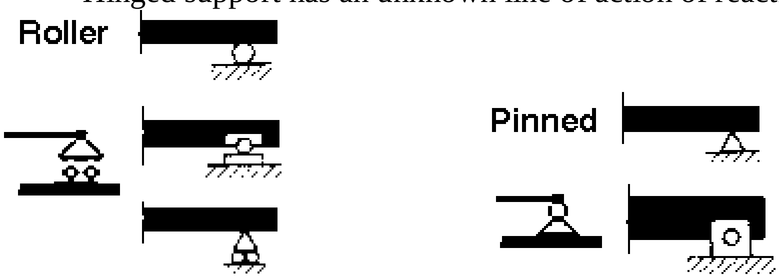
=(7-2)i+(10-4)j+(15-6)k

= **5i+6j+9k**

$$\begin{vmatrix} i & j & k \\ 4 & 6 & 9 \\ 74.28 & 111.42 & 148.56 \end{vmatrix} = \underline{\mathbf{-111.42i-74.28j+111.42k}}$$

47. What are the difference between roller support and hinged support?

Roller support has the known line of action of reaction, always normal to the plane of rollers. Hinged support has an unknown line of action of reaction, at any angle θ with horizontal.



48. Find the unit vector along the force F=2i+3j+5k.

Unit vector is λ

Let,

λ = γf/ IγI

= 2i+3j+5k / √ (22+ (32) + (52))

<u>= 0.324i+0.486j+0.811k</u>

49. **A position vector and force are 2i-3j+4k and 120i-260j+320k respectively. find the moment of the force about the origin.**

<u>Given data</u>:

Position vector is r = 2i-3j+4k

force vector F =120i-260j+320k

<u>Solution:</u>

Moment Mo = r XF,

$$\begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ 120 & -260 & 320 \end{vmatrix} = \underline{\mathbf{80i\text{-}160j\text{-}160k}}$$

50. Write short notes the Varignon's theorem of moment.

The algebraic sum of moments of any number of forces about any point in their plane is equal to moment of their resultant about the same point.

(ie.) F1d1 + F2d2+ F3d3+ - - -- - -- - = R × d

Where,

F1, F2, F3 …etc= Forces in all directions in "N"

D=is distance in "mm"

51. Define the terms Rigid bodies and Moment of force.

Rigid bodies:

Rigid bodies When a body is not subjected to collinear or concurrent force system, then the body is to be idealized as a rigid body.

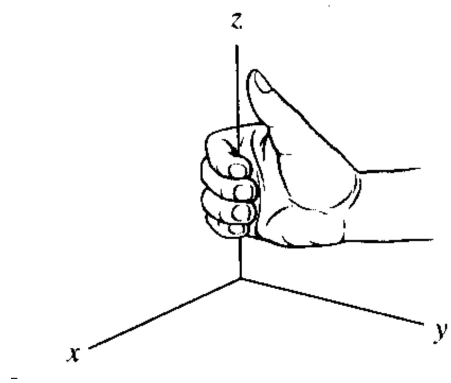
Moment of force:

Moment of force Moment of force is defined as the product of the force and the Perpendicular distance of the line of action of force from the point.It's unit is N-m.

M=F*r (Nm)

52. **Write about the vector (or cross) product**.

The vector product of two intersecting vectors **A** and **B**, by definition, yields a vector **C** that has the magnitude that is the product of the magnitude of vectors **A** and **B** and the sine of the angle between them with a direction that is perpendicular to the plane containing vectors **A** and **B**.



AxB=-BxA=(AB sin(q))e_c

$0<\theta<180^0$

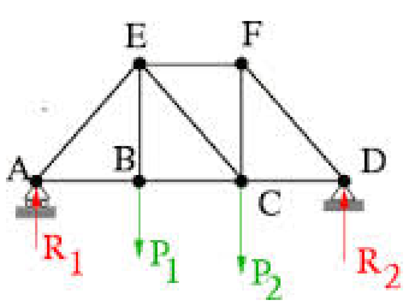
53. Write short notes on Principle of Moments.

The principle of moments, which is also referred to as Varignon's theorem, states that the moment of a force about a point is equal to the sum of the moments of the force's components about the point. Therefore, if the components of a force are known, it may be simpler to determine the moment of each component and then add them.

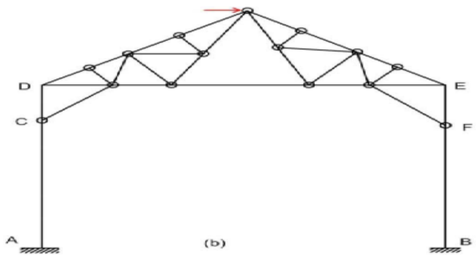
i.e. Mo = r F

54. Write short notes on trusses and frames.(Nov/dec-2008)

<u>Trusses</u>: Structures composed entirely of two force members.



Frames: Structures containing at least one member acted on by forces at three or more points.



Moment of a force along an axis

Couples

55. **Write the objectives of moment of a force along an axis couple**.

Moment of a force along axis couple objectives:

a) Understand the vector formulation for finding the component of a moment along an axis.
b) Understand the idea of a couple and the moment it produces.

56. Write the various tools of moment of a force along an axis couple.

Moment of a force along an axis couple tools:

a) Algebra
b) Position Vectors
c) Unit Vectors
d) Cross Products
e) Dot Products
f) Basic trigonometry **etc.**

57. A simply supported beam AB of length 9 m, carries a uniform distributed ion load of 10 KN/m for a distance of 6 m from leaf end. Calculate the reactions at A and B.

Given data:
Length of the beam: =9m
Rate of U.D.L =10 KN/m
Length of U.D.L = 6m.
Total load due to U.D.L= Length of U.D.L × Rate of U.D.L.
= 6×10=60KN.
To find:
a) The reactions at A and B.

Solution:

We have to calculate all the forces in to concentrated load.

Point load of U.D.L is =60 KN.

The distance of point load from left end is= total distance $\times \frac{1}{2}$

= 6/2=3m.

The two dimensional equilibrium conditions:

i) $\sum HF$ =0, (Say "0" There is no horizontal forces acting on the beam.)

ii) $\sum VF$ =0.

iii) $\sum M$ =0

Let,

ii) $\sum VF$ =0

$R_A+R_B=60$ ---------------1

iii) $\sum M$ =0,

$(R_B\times9)-(6\times10)\times3=0$

$\therefore R_B$ =20KN.

R_B =20KN value substitute in to equation 1.

Then, R_A= 40 KN.

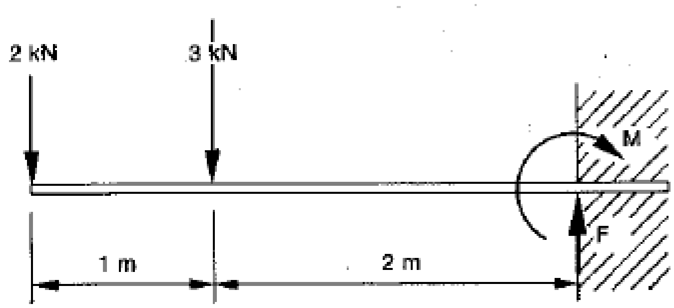
Result:

The reactions of the beam is

R_A= 40 KN.

R_B =20KN.

58. Calculate the reactions of cantilever beam shown in figure. (may/june-2005)



Solution:

The moment equation about the support yields

$\sum M$ **=0**,

M-(3×2)-(2×3)=0

Hence reaction moment M=12 k.N.m.

$\sum VF$ **=0,**

F-3-2=0

Hence the reaction of force F=5 kN.

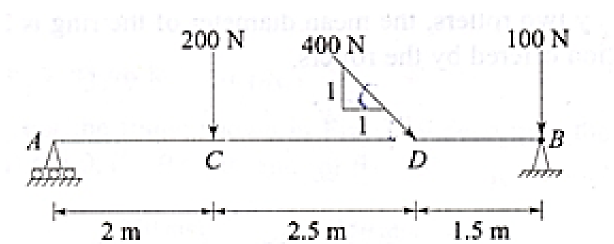
59. Write short notes on position vector.

In geometry, a position or position vector, also known as location vector or radius vector, is a Euclidean vector that represents the position of a point P in space in relation to an arbitrary reference origin O. Usually denoted x, r, or s, it corresponds to the straight line distance from O to P.

i.e r=OP

Part-B

1.**Using the conditions of the equilibrium for a rigid body in two dimensions, determine the reactions for the beam shown in figure which is pinned at B and supported on rollers at A. (16) (Apr/May 2010)**



Solution:

V_A be the Vertical reactions at A. V_B, H_B be the vertical exchange and horizontal reactions at B

Equations of equilibrium are:

$\sum Fx = 0, \sum Fy = 0, \sum m = 0$

400 Cos 45^0-H_B = 0

$\boxed{H_B = 282.84N}$

$\sum Fy = O[\uparrow +]$

$V_A - 200 - 400 \text{ Sin } 45^0 - 100 + V_B = 0$

$V_A + V_B = 582.84$ N

$\sum M = 0$

$-200 \times 2 - 400 \times \text{Sin } 45^0 \times 405 - 100 \times 6 + V_B \times 6 = 0$

$\boxed{V_B = 378.80N}$

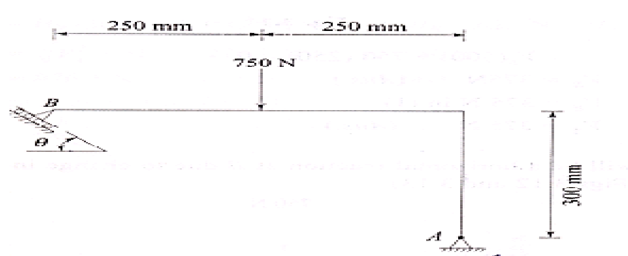
VA + 378.80 = 582.84

$\boxed{V_B = 204.04N}$

Result:

H_B = 282.84N
V_B = 378.8N
V_A = 204.04N

2. For the frame shown in figure determine the reactions at A and B, (a) θ=0 (b) θ=90 (c) θ=30. (16)



Solution:

When $\theta = 0$

$\sum Fx = 0, \sum Fy = 0$ &

$\sum m = 0$

$\sum Fx = 0, [\rightarrow (+)]$

$-H_A = 0$

$H_A = 0$

$\sum Fy = 0$

$V_B - 750 + V_A = 0$

$V_A + V_B = 750$

$\sum m = 0$

$-V_B \times 500 + 750 \times 250 = 0$

V_B= 204.04N

V_A= 204.04N

(b) $\theta = 90°$

$\sum F_x = O[\rightarrow (+)]$

$H_B - H_A = 0 \,; H_A = H_B$

$\sum F_y = 0$

$V_A - 750 = 0; \mathrm{V}_A = 750N$

$\sum m = 0$

$-H_B X 300 + 750 X 250 = 0$

$H_B = 625N = H_A$

$R_A = \sqrt{V_A{}^2 + H_A{}^2} = 976.3N$

$\alpha = 50.2°$

$R_A = 976.3N$

(c) $\theta = 30°$

$\sum m = 0$

$-R_B COS 60° X 300 - R_B SIN 60° X 500 + 750 X 250 = 0$

$R_B = 321.61N$

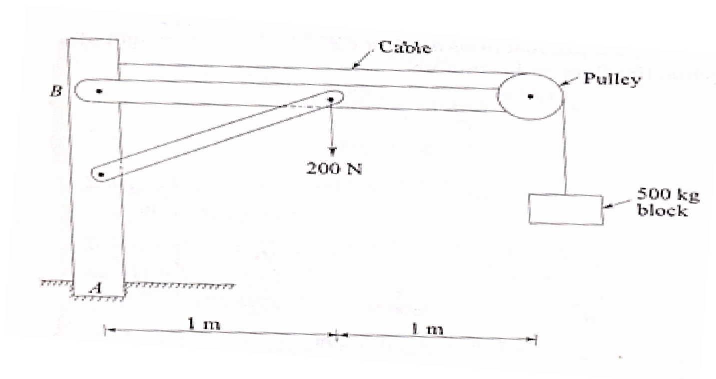
$\sum F_x = 0$

$R_B COS 60° - H_A = 0$

$H_A = 160.81N$

$$\sum F_Y = 0; R_B Sin60° + V_A - 750 = 0$$
$$V_A = 471.48N$$
$$R_A = \sqrt{V_A^{\,2} + H_A^{\,2}}$$
$$R_A = 498.15N$$
$$\alpha = 71.2°$$

3. Determine the reactions at the fixed support A for the loaded frame shown in figure. Take the diameter of the pulley as 250 mm. (16)



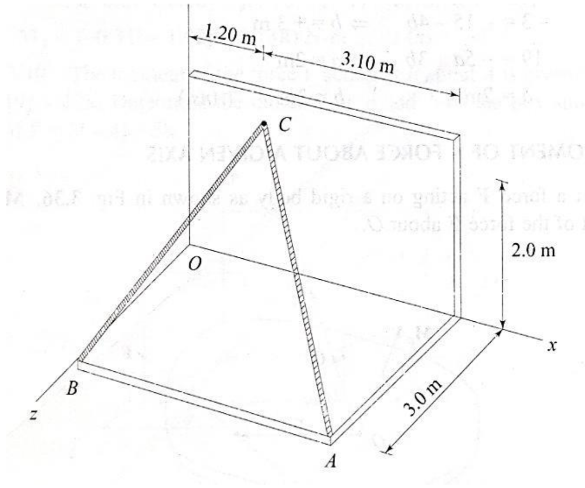
Solution :

$$\sum F_x = 0$$
$$H_A = 0$$
$$\sum F_y = 0$$
$$V_A - 200 - 4905 = 0$$
$$V_A = 5150N$$
$$\sum m = 0$$
$$M_A - 200(1) - 4905(2.125) = 0$$
$$M_A = 10.62KN - m$$

Result:

$$H_A = 0$$
$$V_A = 5150N$$
$$M_A = 10.62KN - m$$

4. The rectangular plate shown in figure is held in the position shown against a vertical wall by a rope passed over a frictionless hook at C. Determine the moment about each of the co-ordinate axes of the force exerted by the rope at A, if the tension in the rope is 1500N. (16) (Nov/Dec-07)



Solution:

$$M_O = r_{A/O} \text{ x F}$$

$$r_{A/O} = xi + yi + zk$$

$$= 4.3i+0j+3k$$

$$F = F\lambda$$

$$\lambda = \frac{AC}{(AC)} = \frac{-3.1i + 2.0j - 3k}{\sqrt{(-3.1)^2 + 2^2 + (-3)^2}}$$

$$= \frac{1}{4.75}(-3.1i + 2j - 3k)$$

$$F = \frac{1500}{4.75}[-3.1i + 2j - 3k]$$

$$= -978.95i + 631.58j - 947.37k$$

$$M_O = \begin{pmatrix} i & j & k \\ 4.3 & 0 & 3 \\ -978.95 & 631.58 & -947.37 \end{pmatrix}$$

$$= i(-1894.74) - j(-1136.84) + k(2715.79)$$

$$= -1894.74i + 1136.84j + 2715.79k$$

Moment of the force about OX axis

$$M_{OX} = i[-1894.74i + 1136.84j + 2715.79k]$$

$$= -1894.74N - m$$

Moment of the force about OY axis

$$M_{OY} = i[-1894.74i + 1136.84j + 2715.79k]$$

$$= 1136.84N - m$$

Moment of the force about OZ axis

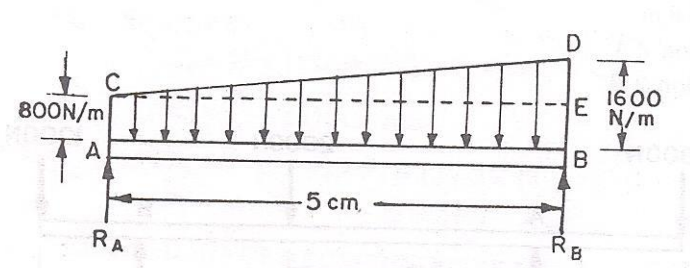
$$M_{OZ} = i[-1894.74i + 1136.84j + 2715.79k]$$

$$= 2715.79N - m$$

RESULT:

$$M_{Ox} = -1894.74N - m$$
$$M_{Oy} = 1136.84N - m$$
$$M_{OZ} = 2715.79N - m$$

5. A simply supported beam of length 5m carries a uniformly increasing load of 800 N/m at one end to 1600 N/m at the other end. Calculate the reactions at both ends. (16)



Solution:

The C.G of the rectangle ABEC will be at 2.5m from A The C.G of the triangle CED will be at 3.33m from A taking moment of all forces about point A.

$$R_B X 5 - (5X800)X2.5 - (1/2X5X800)X(2/3X5) = 0$$
$$5R_B - 1000 - 6666.66 = 0$$
$$R_B = 3333.33N$$

Also foo the equilibrium of the beam, $\sum F_y = 0$

$R_A + R_B =$ Total load on the beam

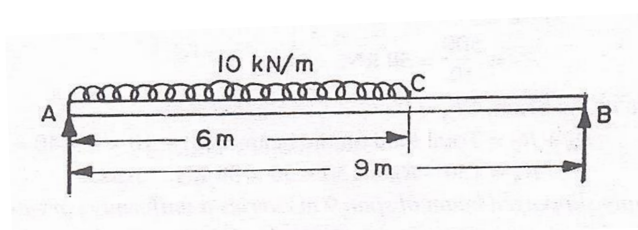
$= 6000$

$R_A = 2666.66N$

Result:

$$R_A = 2666.66N$$
$$R_B = 3333.3N$$

6. A SSB AB of length 9m, carries a UDL of 10 kN/m for a distance of 6m from the left end. Calculate the reactions at A and B. (16)



SOLUTION:

Total load due to UDL = 6 x 10 = **60KN**

60KN will be acting at the middle point od AC i.e at a distance of 6/2 = 3m from A.

Taking the moments of all forces about point A, and equating the resultant moment to zero, we get R_B x 9 – (6 x 10) x 3 = 0

$9R_B - 180 = 0$

R_B = 20 KN

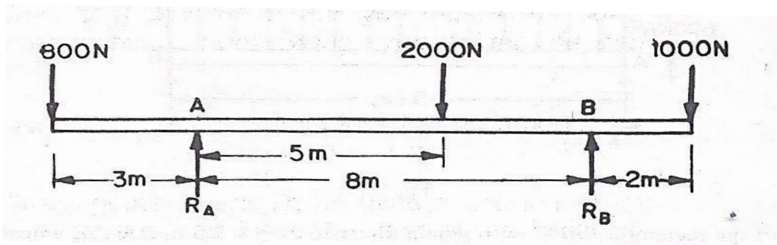
Also for equilibrium

$$\sum F_y = 0$$
$$R_A + R_B = 6X10 = 60$$
$$R_A = 40KN$$

RESULT:

$$R_A = 40KN$$
$$R_B = 20KN$$

7. A beam AB of span 8m, overhanging on both sides, is loaded as shown in figure. Calculate the reactions at the both ends. (16)



Solution :

Taking the moments of all forces about point A

$$R_B X 8 + 800 X 3 - 2000 X 5 - 1000 X (8+2) = 0$$
$$8R_B + 2400 - 10000 - 10000 = 0$$
$$R_B = 220000$$

Also for the equilibrium to the beam,

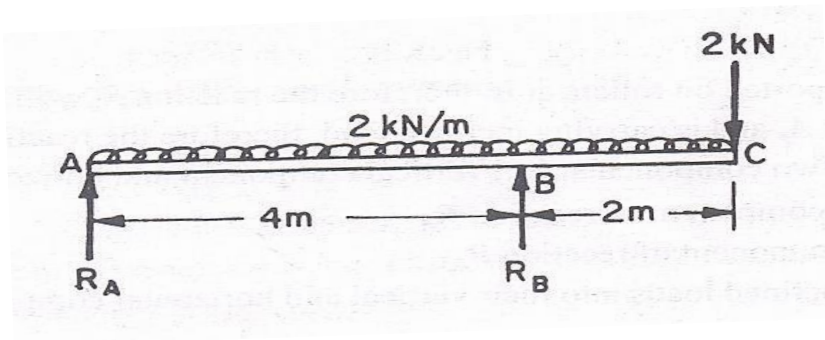
$$R_A + R_B = 800 + 2000 + 1000$$

$$= 3800N$$

Result:

$$R_A = 1600N$$
$$R_B = 2200N$$

8. A beam AB of span 4m, overhanging on one side up to a length of 2m, carries a UDL of 2 kN/m over the entire length of 6m and a point load of 2 kN/m as shown in figure. Calculate the reactions at A and B. (16)



Soulition:

Load due to UDL = 2 x 6 = 12 KN

Load 12KN will act at the middle point of AC at a distance of 3m from A

Taking the moments of all foces about point A and equating the resultant moment to zero.

$$R_B X 4 - (2X6)X3 - 2(4+2) = 0$$
$$4R_B - 36 - 12 = 0$$
$$R_B = 12KN$$

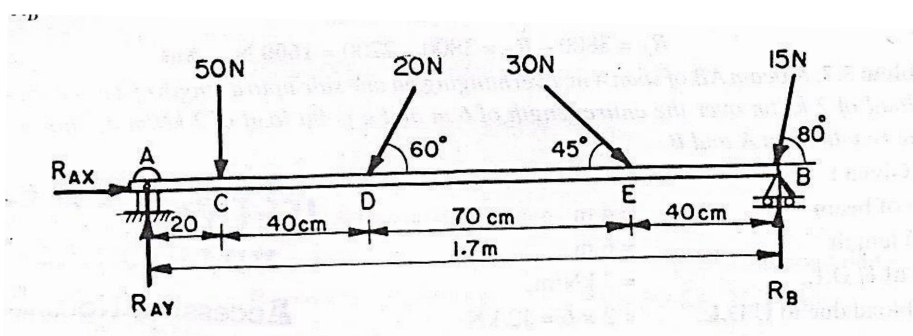
Also for equilibrium

$$\sum F_y = 0$$
$$R_A + R_B = 12 + 2$$
$$= 14$$
$$R_A = 2KN$$

Result:

$$R_A = 2KN$$
$$R_B = 12KN$$

9. **A beam AB 1.7m long is loaded as shown in figure. Determine the reactions at A and B. (16) (Apr/May 2010)**



Solution:

Since the beam is supported on rollers at B,therefore the reactions R_B will be vertical,

The beam is higed at A,and is carrying inclined load, therefore the reactions R_A will be inclined. So it has two component i.e,vertical and horizontal compenents..

Resolving all the inclined loads into vertical and horizontal component

(i) Load at D

Horizontal = $20\cos60^0 = 10N\rightarrow$

Vertical = $20\sin60^0 = 17.32N$

(ii) Load at E

Horizontal = $30\cos45^0 = 21.12N\rightarrow$

Vertical = $30\sin45^0 = 21.2$

(iii) Load at B

Horizontal = $15\sin80^0 = 14.77N\rightarrow$

Vertical = $15\cos80^0 = 2.6N$

For condition of equilibrium, $\sum F_x = 0$

$$R_{AX} - 10 + 12.21 - 2.6 = 0$$

$$R_{AX} = -8.61N$$

-Ve sign shows that assumed direction of R_{AX} is wrong correct direction will be opposite to the direction.

Hence correct direction towards left at A.

$$R_{AX} = 8.61N \leftarrow$$

To find R_B, take moments of all forces about A for equilibrium $\sum M_A = 0$

(50 x 20) + (20sin600) x (20+40) + (30xsin45^0) (20+40+70)+(15sin80^{0}x170)-170R_B = 0

R_B = 42.98N

To find R_{AY}, Apply condition of equilibrium $\sum F_y = 0$

$$R_{AY} + R_B = 50 + 20\sin 60° + 30\sin 45° + 15\sin 80°$$
$$R_{AY} = 103.3 - 42.98$$
$$R_{AY} = 60.32N \uparrow$$

Reaction at A

$$R_A = \sqrt{R_{AX}{}^2 + R_{AY}{}^2}$$

$$= \sqrt{8.61^2 + 60.32^2}$$

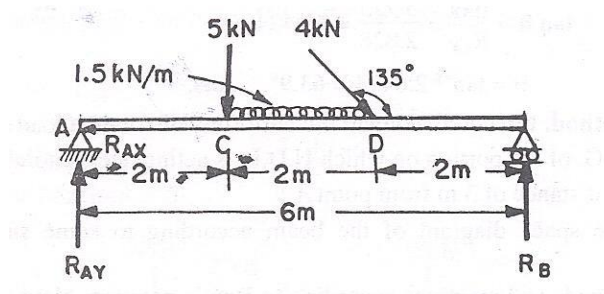
$R_A = 60.92N$

$\tan\theta = \dfrac{R_{AY}}{R_A}$

$= \dfrac{60.32}{8.61}$

$\theta = 81.87°$

10. A beam 6m long is loaded as shown in figure. Determine the reactions at A and B by (a) analytical method, and (b) graphical method. (16)



Solution:

Horizontal component of 4KN at D

$= 4\cos45^0 = 2.828KN\rightarrow$

Vertical component

$= 4\sin450 = 2.828KN\downarrow$

For equilibrium $\sum F_x = 0$

$-R_{AX} + 2.828 = 0$

$R_{AX} = 2.828N$

$\sum M_A = 0$

$R_B X 6 - 5X2 - (2X1.5)(2+2/2) - 4\sin 45°(2+2) = 0$

$\sum F_y = 0$

$R_{AY} + R_B - 5 - (2X1.5) - 4\sin 45° = 0$

$R_{AY} = 5.776KN$

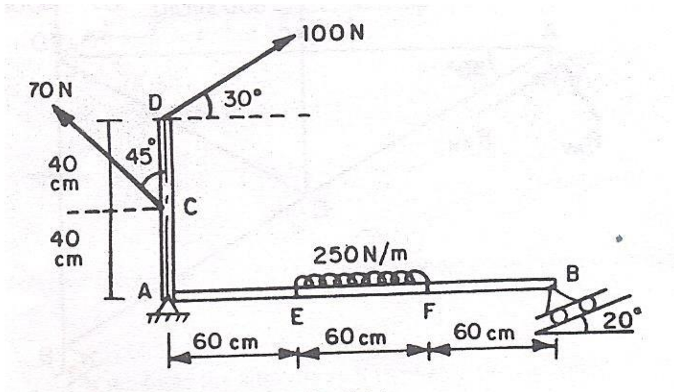
Reaction at A

$R_A = \sqrt{R_{AY}{}^2 + R_{AY}{}^2} = \sqrt{2.828^2 + 5.776^2}$

$R_A = 6.43KN$

$$\tan\theta = \frac{R_{AY}}{R_{AY}} = \frac{5.775}{2.828}$$
$$\theta = 63.9°$$

11. Find the reactions at supports of an L-bent shown in figure. (16)



Solution:

The 1st distance from A on the line of action of R_B = AO

=ABcos20^0

=1.8cos20^0m

For equilibrium of the beam, the moments of all forces about any point should be zero.

Taking moments of all forces about point A, we get

$$100\cos 30° X 80 - 70\sin 45° X 40 + 150 X 90 - R_B X 180\cos 20° = 0$$

6928-1979.6+13500-169.14R_B = 0

R_B = 109.07N

For equilibrium $\sum F_x = 0$

$$R_{AX} + 100\cos 30° - 70\sin 45° - \mathrm{R}_B \sin 20° = 0$$
$$R_{AX} = 0.19N$$

For equilibrium $\sum F_Y = 0$

$$R_{AY} + 100\sin 30° + 70\cos 45° - \mathrm{R}_B\cos 20° = 150$$
$$R_{AY} = 150 - 100\sin 30° - 70\cos 45° - R_B\cos 20°$$
$$R_{AY} = -51.98N$$

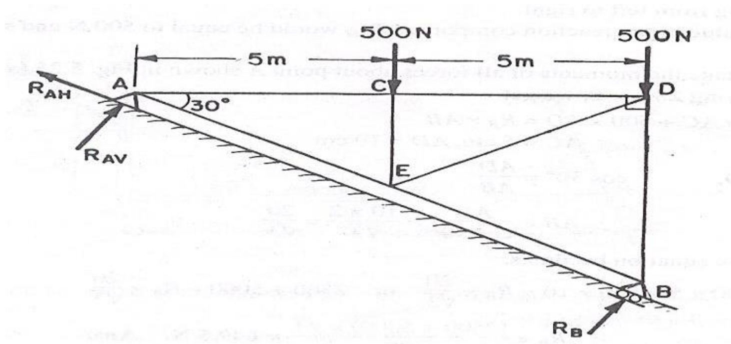
The angle made by R_A with x axis is given by

$$\tan\theta = \frac{R_{AY}}{R_{AX}}$$
$$= \frac{51.98}{0.19}$$
$$\theta = 89.79°$$

Result :

$$R_A = 51.98N$$
$$R_B = 109.07N$$
$$\theta = 89.79°$$

12. Determine the reactions at the hinged support A and the roller support B as shown in figure. (16)



Sum of the components parallel to the inclined surface AB,

$$= 500\sin 30° + 500\sin 30°$$
$$= 500N$$

Acting from left to right

To find R_B, take moments about A

$$500XAC + 500XAD = R_B XAB$$

From Δ ABD, $\cos 30° = \frac{AD}{AB}$

$$AB = \frac{AD}{COS30°} = \frac{20}{\sqrt{3}}$$

About equation becomes

$$500X5 + 500X10 = R_B X\,\frac{20}{\sqrt{3}}$$

$$R_B = 649.6N$$

Equating all the forces $\perp^r$ to the inclined surface AB,

$$R_{AV} + R_B = 500X\cos 30° + 500\cos 30°$$
$$R_{AV} + 649.5 = 1000\cos 30°$$
$$R_{AV} = 216.5N$$
$$R_{AH} = 500N$$

From above $R_A = \sqrt{R_{AH}{}^2 + R_{AV}{}^2}$

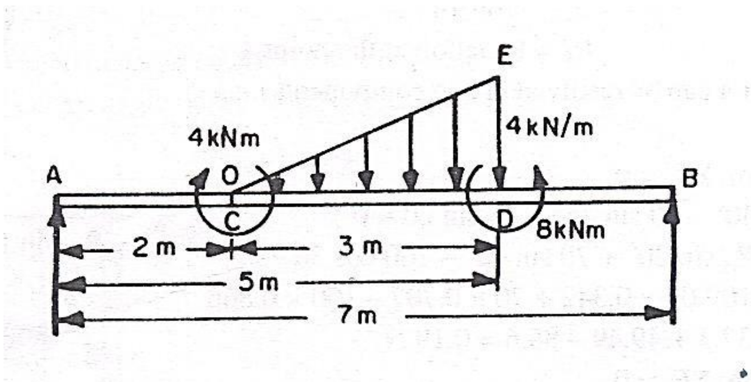
$$= \sqrt{500^2 + 216.5^2}$$

$$R_A = 545N$$

Result:

$$R_A = 545N$$
$$R_B = 649.5N$$

13. A simply supported beam AB of 7m span is subjected to : (1) 4 kN m clockwise couple at 2m from A, (2) 8 kN m anti clockwise couple at 5m from A and (3) a triangular load with zero intensity at 2m from A increasing to 4 kN per m at a point 5m from A. Determine the reactions at A and B. (16)



Solution :

Total load on beam = Area of triangle CDE

= CD x DE / 2

= 3 x 4 / 2 = 6KN

This load will be acting at the C.G of the Δ CDE i.e, at a distance at 2/3 x CD = 2/3 x 3 = 2m

From C or 2 + 2 + 4 on from end A taking the moment of all forces about point A

$$-R_B X 7 + 4 - 8X6X4 = 0$$
$$R_B = \frac{20}{7}KN$$

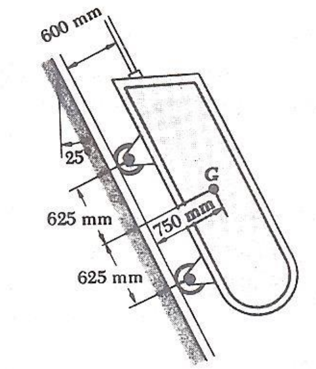
Also for the equilibrium of the beam $\sum F_Y = 0$

$$R_A + R_B = 6KN$$
$$R_A = \frac{22}{7} KN$$

Result

$$R_A = \frac{22}{7} KN$$
$$R_B = \frac{20}{7} KN$$

14. A loading car is at rest on a track forming an angle of 25^0 with the vertical. The combined mass of the car and of its load is 2500 kg, and their center of gravity G is located halfway between the two axles and 750 mm from the track. The car is held by a cable attached 600 mm from the track. Determine the tension in the cable and the reaction at each pair of wheels. (16) (Nov/Dec 2012)



$$W = mg = 2500 X 9.81 = 24.5KN$$

Resolving weight into x & y component

$$W_X = 24.5\cos 25° = 22.5KN$$
$$W_y = -24.5\sin 25° = -10.35KN$$

Equilibrium equations.

Moment about A to eliminate T & R1 $\sum M_A = 0$

$$-10.35 X 0.625 - 22.2 X 0.15 + R_2 X 1.25 = 0$$
$$R_2 = 7.84KN$$

$$\sum M_B = 0$$

$$10.35 X 0.625 - 22.2 X 0.15 - R_1 X 1.25 = 0$$
$$R_1 = 2.51KN$$

The value of T is found by

$$\sum F_x = 0$$
$$22.2KN - T = 0$$
$$T = 22.2KN$$

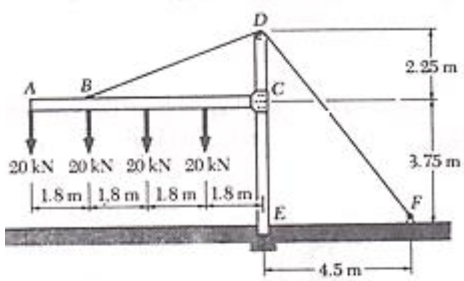
The computations are verified by

$$\sum F_y = 2.51 + 7.84 - 10.35 = 0$$

Resultant:

$$T = 22.2KN$$
$$R_1 = 2.5KN$$
$$R_2 = 7.84KN$$

15. The frame shown supports part of the roof of a small building. Knowing that the tension in the cable is 150 kN, determine the reaction at the fixed end E. (16)



$$DF = \sqrt{4.5^2 + 6^2}$$

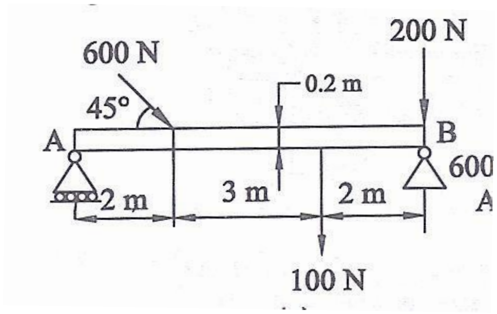
$$= 7.5m$$

$$\sum F_X = 0$$
$$\sum_X + \frac{4.5}{7.5} X150 = 0$$
$$\sum_X = -90KN = 90 \leftarrow$$
$$\sum F_Y = 0$$
$$\sum F_Y - 4X20 - \frac{6}{7.5} X150KN \uparrow$$
$$\sum M_E = 0$$
$$20X7.2 + 20X5.4 + 20X3.6 + 20X1.8 - \frac{6}{7.5} X150X4.5 + M_E = 0$$
$$M_E = 180KN.M = 180.KN.M$$

Result:

$$\sum_x = 90KN \leftarrow$$
$$\sum_y = 200KN \uparrow$$
$$M_E = 180KN$$

16. Determine the horizontal and vertical components of reaction for the beam loaded as shown in figure. Neglect the weight of the beam in the calculations. (16)



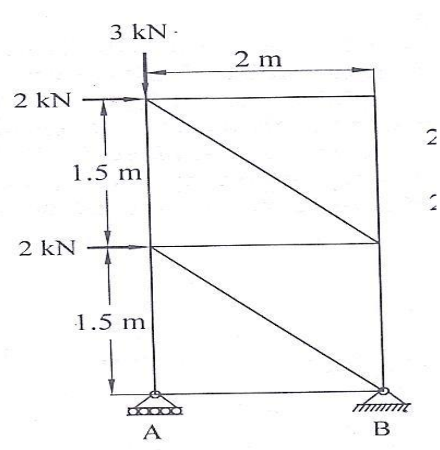
Solution:

$$\sum F_V = 0$$
$$R_A - 600\cos 45° - 100 - 200 + R_{Bv} = 0$$
$$R_A + R_{Bv} = 724.26N$$
$$\sum F_H = 0$$
$$600\cos 45° - R_{BH} = 0$$
$$R_{BH} = 424.26 \leftarrow$$
$$\sum N_B = 0$$
$$-R_A X 7 + 600\cos 45° X 5 - 600\cos 45° X 0.2 + 100 X 2 = 0$$
$$R_A = 319.5N \uparrow$$
$$R_{BV} = 724.26 - 319.5$$
$$R_{BV} = 404.76N$$

Result:

$$R_A = 319.5N$$
$$R_{BH} = 424.26N \leftarrow$$
$$R_{BV} = 404.76N \uparrow$$

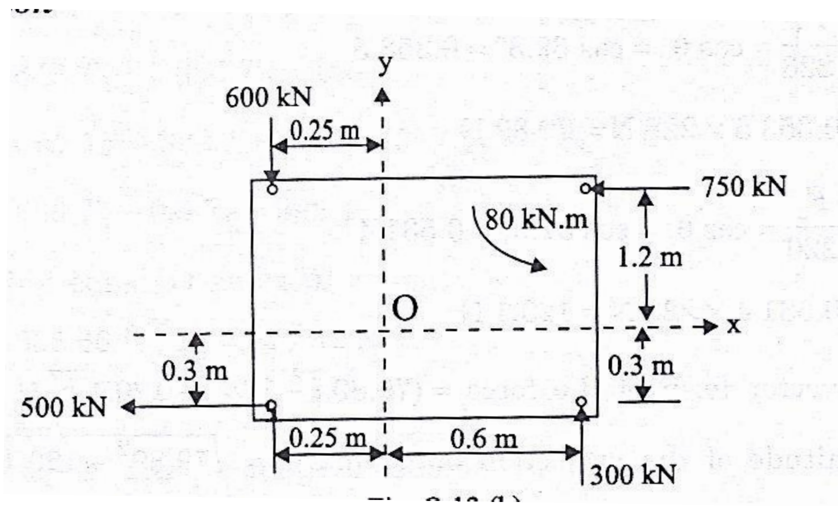
17. Determine the reactions at the supports A and B of the truss shown in figure.(16) (Nov/Dec 2002). (16)



Solution:

$$\sum M_B = 0$$
$$3X2 - 2X3 - 2X1.5 - R_A X2 = 0$$
$$R_A = 1.5KN$$
$$\sum B_V = 0$$
$$R_{BV} = 4.5KN$$
$$\sum F_H = 0$$
$$2 + 2 - R_{BH} = 4KN$$
$$R_{BH} = \sqrt{4^2 + 4.5^2}$$
$$R_{BH} = 6.02KN$$
$$\theta = \tan^{-1}(\frac{4.5}{4})$$
$$\theta = 48.37°$$

18. Four forces and a couple are applied to a rectangular plate as shown in figure. Determine the magnitude and direction of the resultant force couple system. Also determine the distance x from O along X-axis where the resultant intersects. (16) (Apr/May 2004)



Solution:

$$\sum F_X = -750 - 500 = -1250KN$$

$$\sum F_Y = 300 - 600 = -300KN$$

Magnitude of resultant

$$= \sqrt{\sum F_X{}^2 + \sum F_Y{}^2}$$
$$= \sqrt{-1250^2 + (-300)^2}$$
$$= 1285.5KN$$

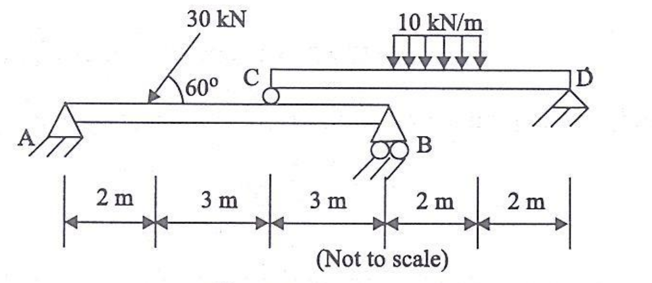
$$\tan\theta = \frac{\sum F_Y}{\sum F_X} = \frac{-300}{-1200}$$
$$\theta = 13.5°$$

$$\sum M_O = 300X0.6 + 750X1.2 + 600X0.25 - 500X0.3 + 80 = 1160KN.m$$

Perpendicular distance of R from O:

$$\frac{\sum M_O}{R} = \frac{11160}{R} = 0.9$$
$$\sin 13.5^o = \frac{0.9}{x}$$
$$x = 3.856m$$

19. Determine the reactions at the supports A,B,C and D for the beam shown in figure. (16) (May/June 2005).



Solution:

Beam CD,

$$M_D = -RCX\,7 + 10X\,2X\,3 = 0$$
$$R_C = 8.57\,\text{KN}\uparrow$$
$$R_C + R_D = 20; R_D = 20 - 8.57 = 11.43\uparrow$$

Beam AB,

The reaction at B is vertical since there is a roller support at B

$$\sum H = 0$$

$$R_{AV} = 30X\cos 60° = 15KN$$

$$\sum V = 0$$

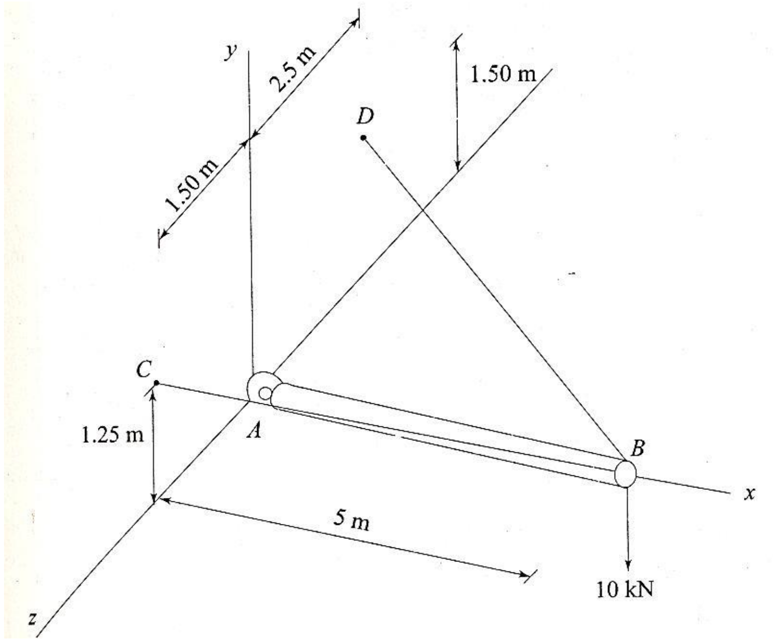
$$R_{AV} - 30\sin 60° - 8.57 + R_B = 0$$

$$\sum M_B = 0$$

$$-R_{AV}X\,8 + 30X\sin 60° X\,6 + 8.57X\,3 = 0$$

$$R_{AV} = 22.7KN$$
$$R_B = 34.55 - 22.7$$
$$R_B = 11.85KN$$

20. Determine the tension in cables BC and BD and the reactions at the ball and socket at A for the rod shown in figure. (16) (Nov/Dec 2009)



Solution:

$$\sum F = 0$$

$$Ax + Ay + Az + T_{BC} + T_{BD} - 10j = 0$$

$$Ax = Axi; Ay = Ayj; Az = Azk$$

$$\overline{T_{BC}} = T_{BC}\lambda BC$$

$$\lambda BC = \frac{\overline{BC}}{BC}$$

$$\lambda BC = \frac{-5i + 1.25j + 1.5k}{\sqrt{(-5)^2 + (1.25)^2 + (1.5)^2}}$$

$$\lambda BC = -0.93i + 0.23j + 0.28k$$

$$T_{BC} = (-0.93i + 0.23j + 0.28k)T_{BC}$$

$$\overline{T_{BC}} = T_{BD}\lambda BD$$

$$\lambda BD = \frac{\overline{BD}}{BD} = \frac{-5i + 1.5j - 2.5k}{\sqrt{(-5)^2 + (1.5)^2 + (-2.5)^2}}$$

$$= -0.86i + 0.26j - 0.43k$$

$$T_{BD} = (-0.86i + 0.26j - 0.43k)T_{BD}$$

From EQ (1):

$$Axi + Ayj + Azk + (-0.93i + 0.23j + 0.28k)T_{BC} + (-0.86i + 0.26j - 0.43k)\,\mathrm{T_{BC}} - 10\mathrm{j} = 0$$

$$(Ax - 0.93T_{BC} - 0.86T_{BD})i + (Ay + 0.23T_{BC} + 0.26T_{BD} - 10)\,\mathrm{j} + (\mathrm{Az} + 0.28T_{BC} - 0.43T_{BD}) = 0$$

$$\sum F_X = O \Rightarrow Ax - 0.93T_{BC} - 0.86T_{BD} = 0$$
$$\sum F_Y = O \Rightarrow Ay + 0.23T_{BC} + 0.26T_{BD} = 10$$
$$\sum F_z = O \Rightarrow Az + 0.28T_{BC} - 0.43T_{BD} = 0$$

Summing the moments of forces about A

$$\sum M_A = 0$$
$$\Rightarrow r_B X (T_{BC} + T_{BD} - 10j) = 0$$
$$\Rightarrow 5iX[(-0.93i + 0.23j + 0.28k)T_{BC} + (-0.86i + 0.26j - 0.43k)T_{BD} - 10j] = 0$$
$$1.15T_{BC}k - 1.4T_{BC}j1.3T_{BD})j + 1.3T_{BD}k + 2.15jT_{BD} - 50k = 0$$
$$(-1.4T_B + 2.15T_{BD})i + (1.15T_{BC} + 1.3T_{BD} - 50)k = 0$$

$$\sum M_Y = 0 \Rightarrow -1.14T_{BC} + 2.15T_{BD} = 0$$
$$\sum M_Z = 0 \Rightarrow 1.15T_{BC} + 1.3T_{BD} = 50$$

Solving at (2) & (6) ;

$$T_{BC} = 25.03KN$$
$$T_{BD} = 16.32KN$$
$$Ax = 37.3KN$$
$$Ay = 0$$
$$Az = 0$$

Unit -3
DISTRIBUTED FORCES

Determination of Areas and Volumes – First moment of area and the Centroid ofsections – Rectangle, circle, triangle from integration – T section, I section, - Anglesection, Hollow section by using standard formula – second and product moments ofplane area – Rectangle, triangle, circle from integration – T section, I section, Anglesection, Hollow section by using standard formula – Parallel axis theorem andperpendicular axis theorem – Polar moment of inertia – Principal moments of inertia ofplane areas – Principal axes of inertia – Mass moment of inertia – Derivation of massmoment of inertia for rectangular section, prism, sphere from first principle – Relation toarea moments of inertia.

Determination of Areas and Volumes – First moment of area and the Centroid of sections

1.**Write about mass and weight.**

In everyday usage, the mass of an object is often referred to as its weight though these are in fact different concepts and quantities. In scientific contexts, mass refers loosely to the amount of "matter" in an object, whereas weight refers to the force experienced by an object due to gravity. In other words, an object with a mass of 1.0 kilogram will weigh approximately 9.81 newton (newton is the unit of force, while kilogram is the unit of mass) on the surface of the Earth (its mass multiplied by the gravitational field strength). Its weight will be less on Mars (where gravity is weaker), more on Saturn, and negligible in space when far from any significant source of gravity, but it will always have the same mass.

2.Write properties of plane surfaces.

i. First moments of area(centroid)
ii. Centre of gravity
iii. Moments of inertia of an area
iv. Product of inertia of an area
v. Area moments of inertia about inclined axes
vi. Principal moments of inertia

3. Define Centroid.

The point at which the total area of plane figure is assumed to be concentrated is known as the centroid of that area. The geometric centroid of a convex object always lies in the object. A non-convex object might have a centroid that is outside the figure itself. The centroid of a ring or a bowl, for example, lies in the object's central void. If the centroid is defined, it is a fixed point of all isometries in its symmetry group. In particular, the geometric centroid of an object lies in the intersection of all its hyperplanes of symmetry. The centroid of many figures (regular polygon, regular polyhedron, cylinder, rectangle, rhombus, circle, sphere, ellipse, ellipsoid, superellipse, superellipsoid, etc.) can be determined by this principle alone.

4.Define Centre of gravity.

Centre of gravity of a body is the point through which the whole weight of the acts. A body is having only one centre of gravity for all position of the body. It is represented by C.G. or simply G. Centre of gravity coincides with the centre of mass if the gravitational forces are taken to be uniform and parallel. The point at which the entire weight of a body may be considered as concentrated so that if supported at this point the body would remain in equilibrium in any position.

5.What is the centroid of length and volume?

a. The centroid of a length is the point at which the entire length is assumed to be concentrated.

b. The centroid of a volume is the point at which the entire volume is assumed to be concentrated.The centroid of volume is the geometric center of a body. If the density is uniform throughout thebody, then the center of mass and center of gravity correspond to the centroid of volume

c. Centroid relates to the length of a line or curve, area of a surface or volume of a body.

6.Write about the central point.

Category of central points

a. Centroid

b. Centre of mass

c. Centre of gravity

Centroid, centre of gravity and centre of mass are concentrated a point in or out of the Object and all are properties of surface.

7.Are centroids and Centre of gravity the same?

The coordinates of thecentre of gravity and the centroid will be the same when two dimensional bodies of uniform thickness and uniform density. But the center of gravity (G) is a point that locates the resultant weight of a system of particles and the centroid (C) is a point which defines the geometric center of an object. Centroid, centre of gravity and centre of mass are concentrated a point in or out of the

Object and all are properties of surface.

8.Is Centre of gravity unique for body?

To be exact, there is no unique centre of gravity. Because, directions of gravitational forces of individual particles of the body are different since they all converge towards the centre of the earth. Further the particle are located at different distances from the earth. Thus the earth gravitational force field is not constant over the body and hence, there is no unique gravity in the strict sense.

9.Write some commonly used centroidal points.

a. Centroid of the lengthof a curve

b. Centroid of the area of a surface

c. Centroid of the volume of a body

d. Mass centre of the mass of a body

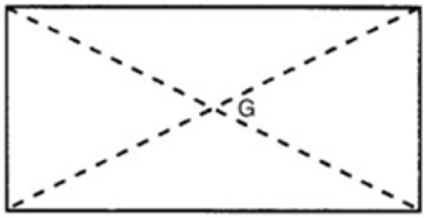
e. Centre of gravity of the gravitational forces acting on a body.

Rectangle, circle, triangle from integration – T section, I section, - Angle section, Hollow section

10. What is axis of symmetry?

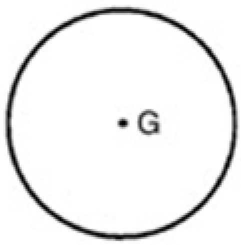
Generally, some composite plane figures have an axis of symmetry. In that occasion, the centroid lies on the point of intersection of symmetrical axis. In the case I section, which is symmetrical about both the axis. T section is symmetric about only one axis and not about other axis. In this case we need to find the location of the centroid about unsymmetrical axis. L section is not symmetrical about both the axis.

11. Draw rectanglular section and locate its centre of gravity.



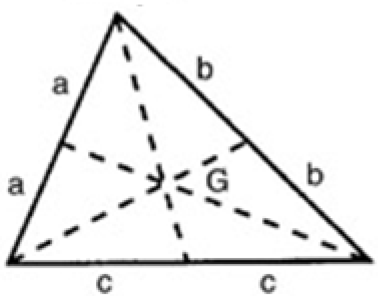
The centroid of a rectangle is the point of intersection of the two diagonals. In the figure point G shows the centre of gravity.

12. Draw circular section and locate its centre of gravity.



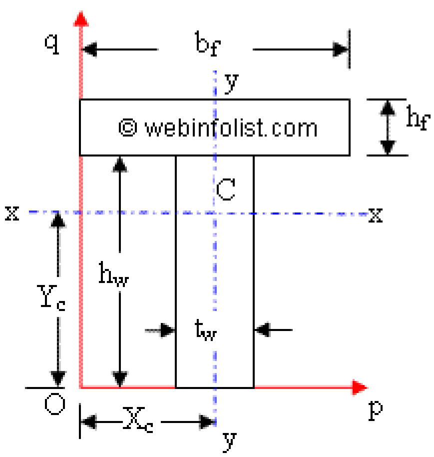
Centre of gravity of a circle and centre of circle is at the same point.

13. Draw triangular section and locate its centre of gravity.



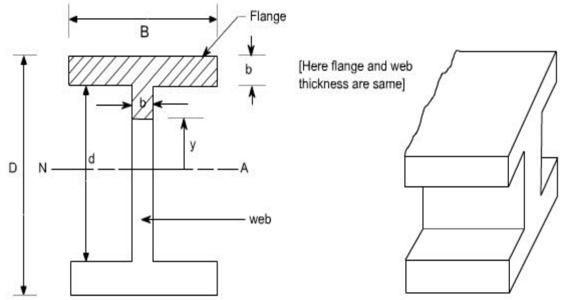
The centroid of a circle is the point at which the three medians of the triangle meet. The median is a line joining the vertex and the middle point of the opposite side.

14. Draw T- section and locate its centre of gravity.



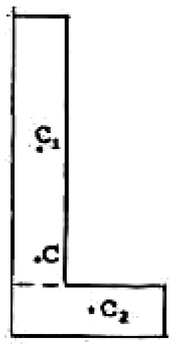
Since the section is symmetrical about yy axis bisecting the web, centre of gravity. The point 'C' is the centre of gravity of T-section.

15. Draw I- section and locate its centre of gravity.



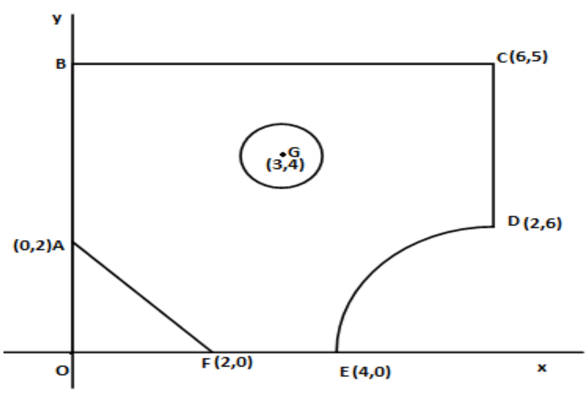
Since the I-section is symmetrical about both xx and yy axis, the centre of gravity lies at the centre of the section.

16. Draw the angle section and locate its centre of gravity?



Since the L-section is not symmetrical about both xx and yy axis, the centre of gravity lies at inside or outside of the object.

17. Draw hollow section and locate its centre of gravity?



Since the hollow-section is not symmetrical about both xx and yy axis, the centre of gravity lies at the point G.

18. Write about centre of gravity of sections with cutout holes.

The centre of gravity of sections with cut out holes is found out by considering the main section, first as a complete one, and then deducting the area of the cut out hole by taking the area of the cut out hole as negative. Let a_1 be the area of the whole section and a_2 be the area of cut out hole then

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

19. <u>Write the Pappus and Guldinius theorems.(Apr/May12)</u>

<u>Theorem I:</u>

The area of a surface of revolution is equal to the product of length of the generating curve and the distance travelled by the centroid of the curve while the surface is being generated.

<u>Theorem II:</u>

The volume of a body or revolution is equal to the product of the generating area and the distance travelled by the centroid of the area while the body is being generated.

20. Find the volume of a sphere of radius r using Pappus-Guldinus theorem.

A solid sphere is obtained by revolving a semicircular area about its diameter.

Volume of solid sphere= area of the semicircle x distance travelled by its C.G.

$$V = \frac{\pi r^2}{2} \times 2 \times \pi \times \frac{4r}{3\pi}$$

$$V = \frac{4}{3}\pi r^3$$

21. How to locate centre of gravity for solid figure?

The centre of gravity of solid bodies such as hemispheres, cylinders, cone etc. is found out in the same way as that of plane figures. The only difference between the plane figure and solid bodies is calculation of volume instead of area.The point at which the entire weight of a body may be considered as concentrated so that if supported at this point the body would remain in equilibrium in any position.

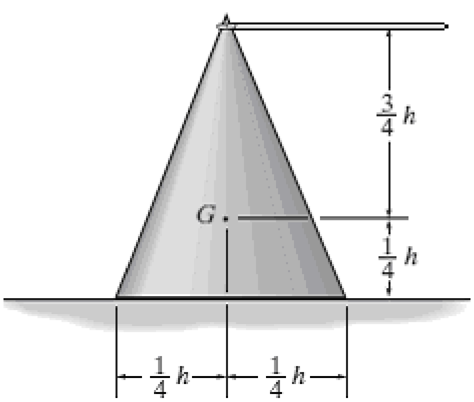
22. A semi-circular are having radius 100 mm is located in the xy plane such that its diametral edge coincide with the y axis. Determine the x coordinate of its centroid.(Apr/May 2003)

$$x = \frac{4r}{3\pi}$$

$$x = \frac{400}{3\pi}$$

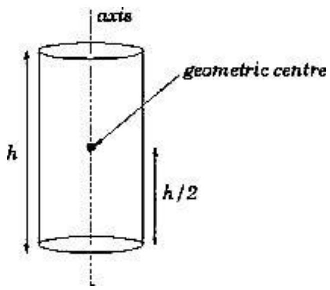
$$x = 42.44mm$$

23. Draw and locate the centre of gravity of cone.



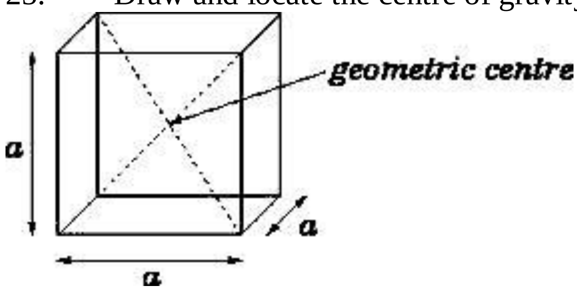
G is the centre of gravity of cone which is ¼ th of height from the base.

24. Draw and locate the centre of gravity of cylinder.



G is the centre of gravity of cylinder which is half of the height from the base.

25. Draw and locate the centre of gravity of cube.



26. Write the formula for finding centroid for solid figure.

Let $X_1, X_2, \ldots, X_n$ are the centroidal distance of $V_1, V_2, \ldots, V_n$ from OY axis and $Y_1, Y_2, \ldots, Y_n$ are the centroidal distance of $V_1, V_2, \ldots, V_n$ from OX axis

$$\bar{x} = \frac{V_1 X_1 + V_2 X_2 + \ldots + V_n X_n}{V_1 + V_2 + \ldots + V_n}$$

$$\bar{y} = \frac{V_1 Y_1 + V_2 Y_2 + \ldots + V_n Y_n}{V_1 + V_2 + \ldots + V_n}$$

Second and product moments of plane area, Parallel axis theorem andperpendicular axis theorem – Polar moment of inertia

27. What is inertia of body?

It is the property by which the body resists any change in its state of rest or of uniform motion. Inertia is the resistance of any physical object to any change in its state of motion, including changes to its speed and direction. It is the tendency of objects to keep moving in a straight line sat constant velocity. The principle of inertia is one of the fundamental principles of classical physics that are used to describe the motion of objects and how they are affected by applied forces.

28. Define moment of inertia.(Nov/Dec 13)

Moment of inertia of a body about any axis is the property by which the body resists rotation about that axis. It is denoted by letters M.I. It is the mass property of a rigid body that determines the torque needed for a desired angular acceleration about an axis of rotation. Moment of inertia depends on the shape of the body and may be different around different axes of rotation. A larger moment of inertia around a given axis requires more torque to increase the rotation, or to stop the rotation, of a body about that axis. Moment of inertia may also be called mass moment of inertia, rotational inertia, polar moment of inertia, or the angular mass.

29. What is second moment of area?

The second moment of area, also known as moment of inertia of plane area, area moment of inertia, or second area moment, is a geometrical property of an area which reflects how its points are distributed with regard to an arbitrary axis. The second moment of area is typically denoted with either an I for an axis that lies in the plane or with a J for an axis perpendicular to the plane. In the field of structural engineering, the second moment of area of the cross-section of a beam is an important property used in the calculation of the beam's deflection and the calculation of stress caused by a moment applied to the beam.

30. Write about second moment of mass.

Moment of inertia of a body is actually the second moment of the mass about the given axis. In the case of plane figures, such as thin, uniform bodies of uniform density, moment of inertia is the second moment of area of the plane figure about the given axis. Strightly speaking, the term moment of inertia should be used only with masses and not areas. However, in practice, the term moment of inertia is used in both the cases of mass and area.

31. Define radius of gyration of an area.

Radius of gyration k is the distance from the axis of rotation to the point where the entire area may be assumed to be concentrated. Radius of gyration or gyradius refers to the distribution of the components of an object around an axis. Most commonly it amounts to the perpendicular distance from the axis of rotation to the centre of mass of a rotating body. The nature of the object does not notionally affect the concept, which applies equally to a surface, a bulk mass, or an ensemble of points.

$$k = \sqrt{\frac{I_x}{A}}$$

32. Write the units of M.I and radius of gyration.(Jan 13)

(i) Unit of M.I. is mm^4 if the area is in mm^2 and the distance is in mm.

(ii) Unit of M.I. is $kg.mm^2$ if the mass is in kg and the distance is in mm.

(iii) Unit of M.I. is $kg.m^2$ if the mass is in kg and the distance is in m.

(iv) Unit of radius of gyration is mm if the unit of M.I. is mm^4

(v) Units of above two may be expresses as cm, km, etc.,

33. What is section modulus?

Section modulus is a geometric property for a given cross-section used in the design of beams or flexural members.Though generally section modulus is calculated for the extreme tensile or compressive fibres in a bending beam, often compression is the most critical case due to onset of flexural torsional buckling. Iis the second moment of area (or moment of inertia) and y is the distance from the neutral axis to any given fibre.

Section modulus, $z = \frac{I}{Y}$

34. State the parallel axis theorem.(Apr/May 11)

It states that, " if the moment of inertia of a plane area about an axis through its centre of gravity be denoted by I_G, then the moment of inertia of area about axis AB parallel to the first and at a distance 'h' from thecentre of gravity is given by

$$I_{AB}=I_G+Ah^2$$

It can be used to determine the mass moment of inertia or the second moment of area of a rigid body about any axis, given the body's moment of inertia about a parallel axis through the object's center of mass and the perpendicular distance between the axes.

35. State the perpendicular axis theorem.

It states that moment of inertia of a plane lamina about an axis perpendicular to the lamina and passing through its centroid is equal to the sum of the moment of inertias of the lamina about two mutually perpendicular axes passing through the centroid and in the plane of lamina of area 'A' with xx, yy and zz axes passing through the centroid G.

$I_{zz}=I_{xx}+I_{yy}$

36. Define polar moment of inertia.

Polar moment of area is a quantity used to predict an object's ability to resist torsion, in objects (or segments of objects) with an invariant circular cross section and no significant warping or out-of-plane deformation.[1] It is used to calculate the angular displacement of an object subjected to a torque. It is analogous to the area moment of inertia, which characterizes an object's ability to resist bending and is required to calculate displacement.

$I_{zz}=J=I_{xx}+I_{yy}$

The larger the polar moment of area, the less the beam will twist, when subjected to a given torque.

37. Write the moment of inertia formula for rectangular section.

Let I_{xx} and I_{yy} be the moment of inertia of the rectangular block about the xx axis and yy axis.

$I_{xx}=\frac{bd^3}{12}$

$I_{yy}=\frac{db^3}{12}$

Where, b- breadth and d- depth

38. Write the moment of inertia formula for triangular section.

Let I_{xx} and I_{yy} be the moment of inertia of the triangular section about the xx axis and yy axis.

Moment of inertia about the base(AB)

$I_{AB}=\frac{bh^3}{12}$

Moment of inertia about centroidal axis

$I_{xx}=\frac{bh^3}{36}$

Where, b- breadth and h- height

39. Write the moment of inertia formula for circular section.

Let I_{xx} and I_{yy} be the moment of inertia of the circular section about the xx and yy axis.

$I_{yy}=I_{xx}=\frac{\pi r^4}{4}$

Where, r-radius of circle

40. Write the moment of inertia formula for semi-circular section.

Let I_{xx} and I_{yy} be the moment of inertia of the block about the xx and yy axis.

$I_{xx}= 0.11\ r^4$

$I_{yy}= \dfrac{\pi d^4}{128}$

Where, d and r are the radius and diameter of circle.

41. Find the radius of gyration of a rectangular area of MI about its base $9x10^4\ cm^4$ and cross sectional area 300 cm^2.(May/June 2013)

$$k = \sqrt{\frac{I}{A}}$$

$$k = \sqrt{\frac{90000}{300}}$$

$$k = 17.32cm$$

Principal moments of inertia ofplane areas – Principal axes of inertia – Mass moment of inertia

42. Define principal moment of inertia.

The axis about which moments of inertia are maximum and minimum are known as principal axes. When these two axes are passing through centroid of area it is known as centroidal principal axis. Now the maximum and minimum moments of inertia are called principal moments of inertia. One of the major interest in the moment of inertia of area is determining the orientation of the orthogonal axes passing a pole on the area with maximum or minimum moment of inertia about the axes.

43. Define principal axes.(Apr/May 2004)

The axis about which moments of inertia are maximum and minimum are known as principal axes. When these two axes are passing through centroid of area it is known as centroidal principal axis. If an object has an axis of symmetry, that axis isalways a principal axis. Now the maximum and minimum moments of inertia are called principal moments of inertia.

44. When will the product of inertia of a lamina become zero?(Nov/Dec 2011)

The product of inertia is zero when one or both of the x-x and y-y axes, happen to the axes of symmetry. Because for each element dA of co-ordinates x and y, there is an element with coordinates x and y or –x and y thus making the integral

$$\int xydA = 0$$

45. What is mass moment of inertia?

The mass moment of inertia is one measure of the distribution of the mass of an object relative to a given axis. The mass moment of inertia is denoted by I and is given for a single particle of mass m. When calculating the mass moment of inertia for a rigid body, one thinks of the body as a sum of particles, each having a mass of dm. Integration is used to sum the moment of inertia of each dm to get the mass moment of inertia of body.

46. What is meant by product of inertia?

Relative to two rectangular axes, the sum of the products formed by multiplying the mass (or, sometimes, the area) of each element of a figure by the product of the coordinates corresponding to those axes.The values of the products of inertia depend on the orientations of the coordinate axes. For every point of the body or system, there exist at least three mutually perpendicular axes, called the principal axes of inertia, for which the products of inertia are equal to zero.

47. How do you express radius of gyration in terms of mass moment of inertia?

Radius of gyration or gyradius refers to the distribution of the components of an object around an axis. In terms of mass moment of inertia, it is the perpendicular distance from the axis of rotation to a point mass(of mass, m) that gives an equivalent inertia to the original object(s). The nature of the object does not notionally affect the concept, which applies equally to a surface, a bulk mass, or an ensemble of points.

48. State the parallel axis theorem as applied to mass moment of inertia.

The moment of inertia around any axis can be calculated from the moment of inertia around parallel axis which passes through the center of mass. The equation to calculate this is called the parallel axis theorem. The mass moment of inertial should not be confused with the area moment of inertia which has units of length to the power four.Mass moments of inertia naturally appear in the equations of motion, and provide information on how difficult (how much inertia there is) it is to rotate the particle around given axis.

Derivation of massmoment of inertia for rectangular section, prism, sphere from first principle Relation toarea moments of inertia.

49. Write the use of mohr's circle in mass moment of inertia.

i. If we plot I_{xx} or I_{yy} in X-axis and I_{xy} in Y axis for different angle of ϴ then the graph forms a circle which is known as Mohr's circle.

ii. It gives moment of inertia about the principal axes one is maximum value another is minimum.

iii. Mohr's circle used in strength of material to find the principal stresses

50. Write the steps to find the principal moment of inertia of unsymmetrical section.

i. given object is split into convenient parts.

ii. find out the centre of gravity of given objects

iii. find out the product moment of inertia about the centroidal axis I_{xy}

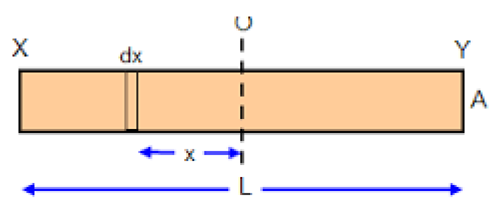
iv. find out the inclination

v. finally calculate principal moment of inertia using required formula

51. Differentiate mass moment of inertia and area moment of inertia.

Area moment of inertia	Mass moment of inertia
Masses are assumed to be concentrated and hence area of plane figures are taken to find the moment of inertia	It is consider only for solid figure
Radius of gyration of plane figure about any axis is defined as the distance at which the entire area A of the figure is assumed to be concentrated, such that the moment of inertia of area A about the axis is same	Radius of gyration of solid figure about any axis is defined as the distance at which the entire mass M of the figure is assumed to be concentrated, such that the moment of inertia of mass M about the axis is same

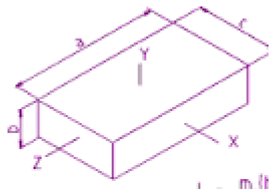
52. Write the formula for mass moment of inertia of thin uniform rod



$$I_{zz} = \frac{1}{12} m(d^2 + L^2)$$

Where, d- diameter
L-Length
m- mass

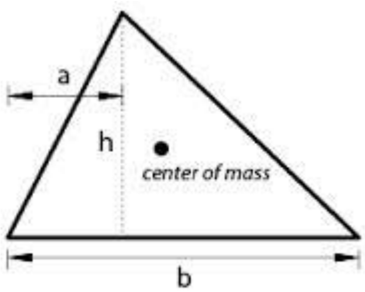
53. Write the formula for mass moment of inertia of rectangular plate.



$$I_{zz} = \frac{1}{12} m(h^2 + b^2)$$

Where, b- breath
h-height
m- mass

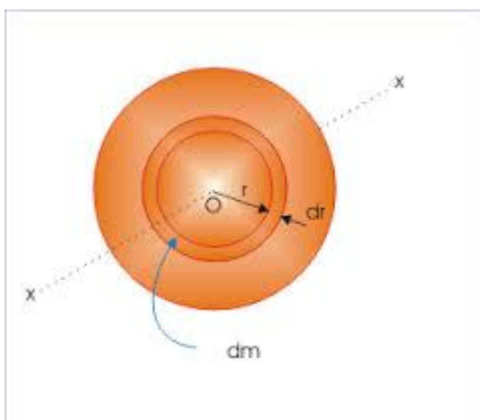
54. Write the formula for mass moment of inertia oftriangular plate.



$$I_{BC} = \frac{mh^2}{6}$$

Where, h-height
m-mass

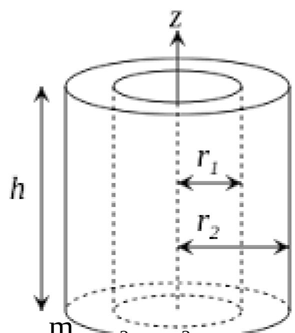
55. Write the formula for mass moment of inertia ofcircular plate.



$$I_{zz} = \frac{mR^2}{6}$$

Where, m-mass
R- radius of circular plate

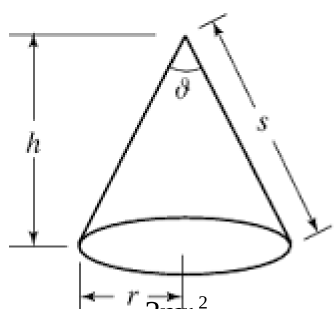
56. Write the formula for mass moment of inertia ofcylinder.



$$I_{zz} = \frac{m}{6}(3r^2 + h^2)$$

Where, m-Mass
r- Radius of cylinder
h- Height of cylinder

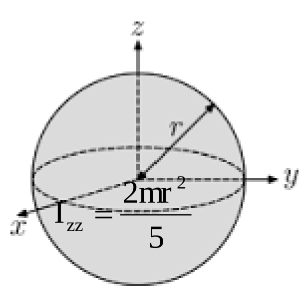
57. Write the formula for mass moment of inertia ofcone.



$$I_{axis} = \frac{3mr^2}{10}$$

Where, m-Mass
r- Radius of cone
h- Height of cone

58. Write the formula for mass moment of inertia forsphere



$$I_{zz} = \frac{2mr^2}{5}$$

Where, m-Mass
r- Radius of sphere

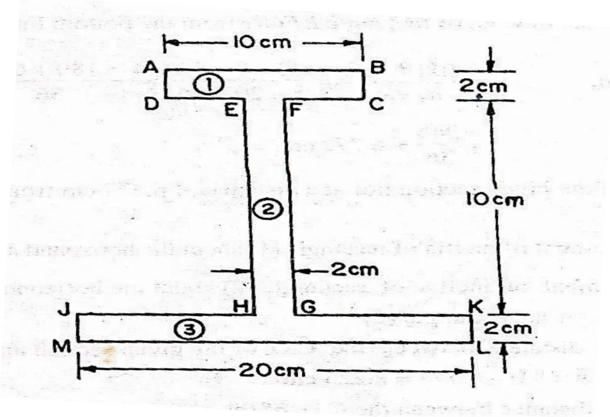
59. What is composite section?

Composite Construction of Buildings refers to any members composed of more than one material. The parts of these composite members are rigidly connected such that no relative movement can occur. Composite beams are normally hot rolled or fabricated steel sections that act compositely with the slab. The composite interactionis achieved by the attachment of shear connectors to the top flange of the beam.

60. Write the steps to be followed while determining the MI of composite bodies.
i. Identify the components of the composite body
ii. Recollect the MI of the individual components about their centroidal axes.
iii. Calculate the MI of the individual components about the required axis by applying the parallel axis theorem.
iv. Find the sum of the individual components to obtain the MI of the composite body.

PART – B

1.Find the moment of inertia of the section shown in figure about the centroidal axis X-X perpendicular to the web.(16) (Apr/May 2004)



Solution:

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$\bar{y}$ - distance of the C.G of the section from the bottom line ML

a_1 – area of the rectangle ABCD = 10*2 = 20 cm^2

y_1 – distance of the C.G of the rectangle ABCD from the bottom line ML.

= 2+10+(2/2)

=12+1 = 13 cm

a_2 – area of the rectangle EFGH = 10*2 = 20cm^2

y_2 - distance of the C.G of rectangle EFGH from the bottom line ML

= 2 + (10/2) = 7 cm.

a_3 – area of the rectangle JKLM = 20*2 = 40cm^2

y_3 - distance of the C.G of rectangle JKLM from the bottom line ML

= (2/2) = 1 cm.

$$\bar{y} = \frac{(20*13)+(20*7)+(40*1)}{20+20+40}$$

$$\bar{y} = 5.5cm$$

Moment of inertia about horizontal axis passing through C.G.

IG_1, IG_2 and IG_3 – moment of inertia of rectangle 1, 2 and 3 respectively.

h_1 – distance between C.G of rectangle I and C.G of given section.

= y_1 - $\bar{y}$ = 13 – 5.5 = 7.5 cm

h_2 = y_2 - $\bar{y}$ = 7 – 5.5 = 1.5 cm

h_3= $\bar{y}$ - y_3 = 5.5 – 1 = 4.5 cm

Now,

$$IG_1 = \frac{10*2^3}{12} = 6.667cm$$

$$IG_2 = \frac{2*10^3}{12} = 166.66cm$$

$$IG_3 = \frac{20*2^3}{12} = 13.33cm$$

Moment of inertia of rectangle 1

$= IG_1 + a_1h_1^2 = 6.667 + (20*7.5^2) = 1131.667\ cm^4$

Moment of inertia of rectangle 2

$= IG_2 + a_2h_2^2 = 166.667 + (20*1.5^2) = 211.667\ cm^4$

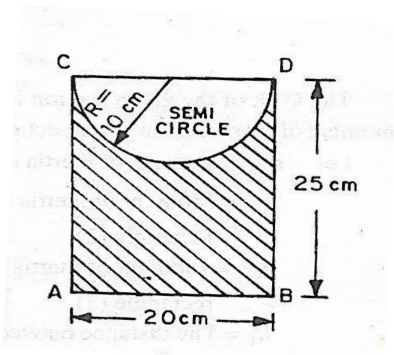
Moment of inertia of rectangle 3

$= IG_3 + a_3h_3^2 = 13.33 + (40*4.5^2) = 823.33\ cm^4$

Moment of inertia of the given section

$= 1131.667 + 211.667 + 823.33 = 2166.667\ cm^4$.

2. Find the moment of inertia of the area shown shaded in figure about edge AB. (16) (Apr/May2004)



Solution:

$$\begin{bmatrix}\text{moment of inertia of} \\ \text{the shaded portion about AB}\end{bmatrix} = \begin{bmatrix}\text{moment of inertia of} \\ \text{rectangle ABCD about AB}\end{bmatrix} - \begin{bmatrix}\text{moment of inertia of} \\ \text{semi circle on DC about AB}\end{bmatrix}$$

Moment of inertia of ABCD about AB

$$= \frac{bd^3}{3} = \frac{20*25^3}{12} = 104.167cm^4$$

Moment of inertia of semi circle about DC

$$= \frac{1}{2}[momentofinertiaofacircleofradius10cmaboutadiameter]$$

$$= \frac{1}{2}\left[\frac{\pi}{4}d^4\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4}20^4\right]$$

$$= 3.925cm^4$$

Distance of C.G of semi circle from DC

$$= \frac{4r}{3\pi}$$

$$= \frac{4*10}{3\pi} = 4.24cm$$

Area of semi circle = $\frac{\pi r^2}{2} = \frac{\pi*10^2}{2} = 157.1cm^2$

$$\begin{bmatrix}\text{Moment of inertia of semi circle}\\ \text{about a line through its C.G parallel to CD}\end{bmatrix}=\begin{bmatrix}\text{moment of inertia of}\\ \text{semi circle about CD}\end{bmatrix}-\left[area*dis\tan ceofC.Gof\sec tionfromDC\right]$$

$= 3925 – 157.1 * 4.24^2$

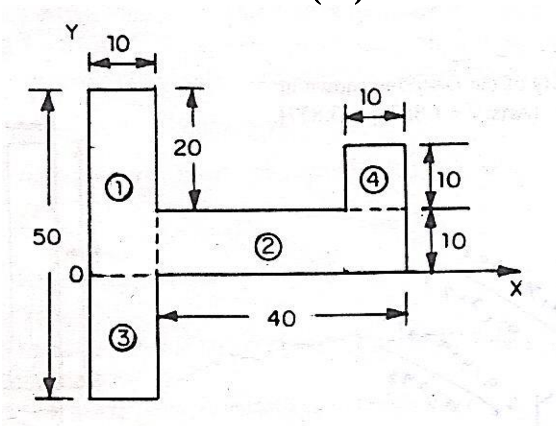
MI = 1100.72 cm^4

Distance of C.G of semi circle from AB = 25 – 4.24 = 20.76 cm

M.O.I of semi circle about AB = 1100.72 + 157.1*20.762 = 68807.3 cm^4

M.O.I of shaded portion about AB = 104.167 – 68807.30 = 35359.7 cm^4

3.Locate the C.G. of the area shown in figure with respect to co-ordinate axes. All dimensions are in mm.(16)



Solution:

a_1 = 10*30 = 300 mm^2, x_1 = 5mm, y_1 = 15mm

a_2 = 40*10 = 400 mm^2, x_2 = 10+20 = 30 mm, y_2 = 5 mm

a_3 = 10*20 = 200 mm^2, x_3 = 5 mm, y_3 = -100 mm

a_4 = 10*10 = 100 mm^2, x_4 = 45 mm, y_4 = 10+5 = 15 mm

$$\bar{x}=\frac{a_1x_1+a_2x_2+a_3x_3+a_4x_4}{a_1+a_2+a_3+a_4}$$

$$\bar{x}=\frac{1500+12000+1000+4500}{1000}$$

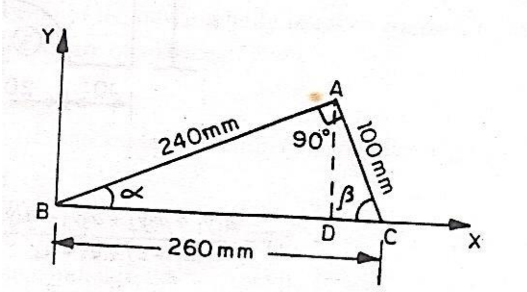
$$\bar{x}=19mm$$

$$\bar{y}=\frac{a_1y_1+a_2y_2+a_3y_3+a_4y_4}{a_1+a_2+a_3+a_4}$$

$$\bar{y}=\frac{4500+2000-2000+1500}{1000}$$

$$\bar{y}=6mm$$

4. A thin homogeneous wire is bent into a triangular shape ABC such that AB=240mm, BC=260mm and AC=100. Locate the C.G. of the wire with respect to the co-ordinate axes. Angle at A is right angle. (16)



Solution:

$$\frac{BC}{\sin 90^0} = \frac{AC}{\sin\alpha} = \frac{AB}{\sin\beta}$$

$$\sin\alpha = \frac{AC\sin 90^0}{\mathrm{BC}} = \frac{100}{260}$$

$$\alpha = 22.62^0$$

$$\sin\beta = \frac{AB\sin 90^0}{\mathrm{AC}} = \frac{240}{260}$$

$$\beta = 67.38^0$$

$$\bar{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3}{L_1 + L_2 + L_3}$$

L_1 = AB = 240mm

$$x_1 = \frac{240}{2}\cos\alpha = 120*\cos 22.62^0 = 110.77$$

L_2 = BC = 260mm

$$x_2 = \frac{260}{2}\cos 90^0 = 130*\cos 90^0 = 130$$

L_3 = AC = 100mm

$$x_3 = BD + \frac{100}{2}\cos\beta = 240*\cos 22.62^0 + 50\cos 67.38^0 = 240.77$$

$$\bar{x} = \frac{240*110.77 + 260*130 + 100*240.77}{240 + 260 + 100}$$

$$\bar{x} = 140.77mm$$

$$\bar{y} = \frac{L_1y_1 + L_2y_2 + L_3y_3}{L_1 + L_2 + L_3}$$

$$y_1 = \frac{240}{2}\sin\alpha = 120*\sin 22.62^0 = 46.154$$

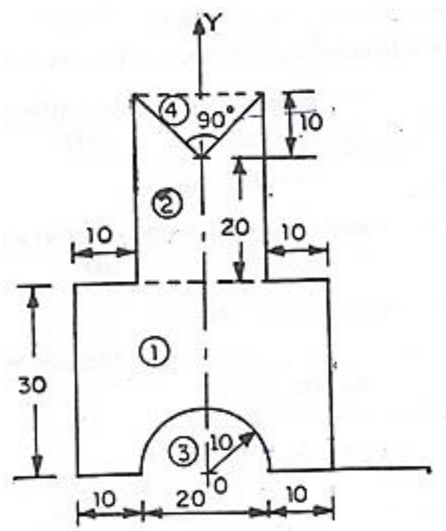
$$y_2 = \frac{260}{2}\sin 90^0 = 0$$

$$y_3 = \frac{100}{2}\sin\beta = 50\sin 67.38^0 = 48.154$$

$$\bar{y} = \frac{240*46.15 + 260*0 + 100*46.15}{600}$$

$$\bar{x} = 26.154mm$$

5.Determine the C.G. of the plane lamina shown in figure. All dimensions are in mm. (16)



Solution:

Figure is symmetrical about yy axis

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4}{a_1 + a_2 + a_3 + a_4}$$

a_1 = 40*35 = 1200 cm^2, y_1 = 30/2 = 15 cm

a_2 = 30*20 = 600 cm^2, y_2 = 30+(30/2) = 45 cm

a_3 = -π*102/2 = -50π cm^2, y_3 = 4r/3π = 40/3π

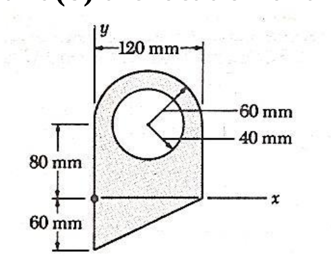
a_4 = -20*10/2 = -100cm^2, y_4 = 60-10/3 = 170/3

$$\bar{y} = \frac{1200*15 + 600*45 - 50\pi*\frac{40}{3\pi} - 100*\frac{170}{3}}{1200 + 600 - 50\pi - 100}$$

$$\bar{y} = \frac{38666.66}{1542.92}$$

$$\bar{y} = 25.06cm$$

6.For the plane area shown, determine (a) the first moment with respect to the x and y axes and(b) the location of the centroid.(16)



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Component	Area	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
Rectangle	120*80 = 9.6 * 10^3	60	40	576*10^3	384*10^3
Triangle	½(120*60)=3.6*10^3	40	-20	144*10^3	-72*10^3
Semi – circle	½(π*60^2)=5.65*10^3	60	105.46	339.3*10^3	596.4*10^3
Circle	-π*40^2=-5.027*10^3	60	80	-301.6*10^3	-402.2*10^3
	ΣA=13.82*10^3			$\sum \bar{x}A = 757*10^3$	$\sum \bar{y}A = 506.2*10^3$

i. first moment of area: $Q_x = \Sigma \bar{y}A = 506.2*10^3\ mm^3$

$Q_y = 757.7*103 =$ $\sum \bar{x}A = 758*10^3 mm^3$

ii. location of centroid:

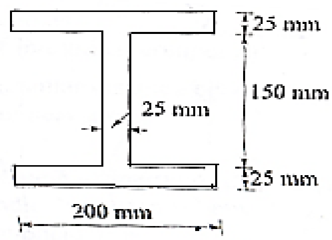
$$\bar{x}\sum A = \sum \bar{x}A; \bar{x}(13.825*10^3) = 757.7*10^3$$

$$\bar{x} = 54.8mm$$

$$\bar{y}\sum A = \sum \bar{y}A; \bar{y}(13.825*10^3) = 506.2*10^3$$

$$\bar{x} = 36.6mm$$

7.Find the moment of inertia about the centroidal axes of the I-section shown in figure.(16)



Solution:

$$\bar{x} = \frac{200}{2} = 100mm$$

$$\bar{y} = \frac{200}{2} = 100mm$$

MI of the component area (1) about xx-axis = $(I_{self})_{xx} + a(p)^2$

$$I_{self} = \frac{bd^3}{12} = \frac{200*25^3}{12} = 260.42*10^3 mm^4$$

$a = 200*25 = 5000mm^2$, $y_1 = 200 - 12.5 = 187.5$ mm

$p = \bar{y} - y_1 = 100 - 187.5 = -87.5$ mm

M. $I_{CD} = (260.42 * 10^3) + (5000*-87.5^2) = 38.54 * 10^6\ mm^4$

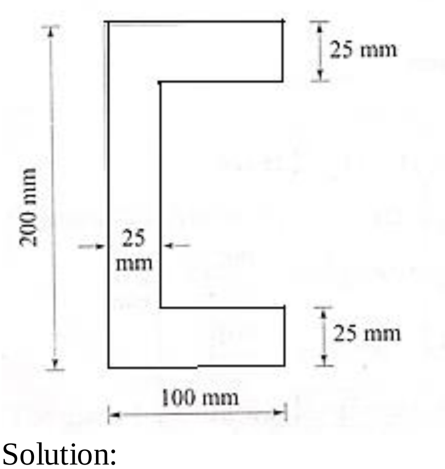
M.I of (2). Here M. I reduces to $(I_{self})xx$ alone, since the xx – axis passes through its C.G.

For (3) = $(I_{self})_{xx} + ap^2$

$$I_{self} = \frac{bd^3}{12} = \frac{200*25^3}{12} = 260.42*10^3 mm^4$$

8.Determine moment of inertia about the centroidal axes for the channel shown in figure.(16)

(Apr/May 2010)



Solution:

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

A_1 = 100 x 25 = 2500 mm2

x_1 = 100/2 = 50 mm

A_2 = 150 x 25 = 3750 mm2

x_2 = 25/2 = 12.5 mm

A_3 = 100 x 25 = 2500 mm2

x_3 = 100/2 = 50 mm

$$\bar{x} = \frac{(2500x50) + (3750x12.5) + (2500x50)}{2500 + 3750 + 2500}$$

$$\bar{x} = 33.93mm$$

$$\bar{y} = 200/2 = 100mm$$

M.I about xx axis

$$I_{xx} = \left[\frac{b_1d_1^{\ 3}}{12} + b_1d_1(\bar{y} - y_1)^2 + \left[\frac{b_2d_2^{\ 3}}{12}\right] + \frac{b_3d_3^{\ 3}}{12} + b_3d_3(\bar{y} - y_3)^2\right]$$

$$I_{xx} = \left[\frac{100x25^3}{12} + 100x25(100 - 187.5)^2 + \left[\frac{25x150^3}{12}\right] + \frac{100x25^3}{12} + 100x25(100 - 12.5)^2\right]$$

$$I_{xx} = 45.57x10^6 mm^4$$

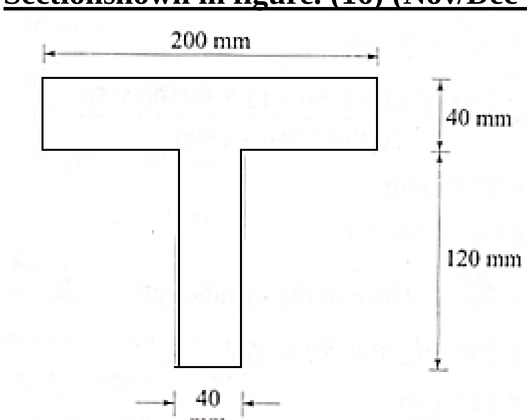
M.I about yy axis

$$I_{yy} = \left[\frac{d_1b_1^{\ 3}}{12} + b_1d_1(\bar{x} - x_1)^2 + \left[\frac{d_2b_2^{\ 3}}{12}\right] + \frac{d_3b_3^{\ 3}}{12} + b_3d_3(\bar{x} - x_3)^2\right]$$

$$I_{yy} = \left[\frac{25x100^3}{12} + 100x25(33.93 - 50)^2 + \left[\frac{150x25^3}{12}\right] + \frac{25x100^3}{12} + 100x25(33.93 - 50)^2\right]$$

$$I_{yy} = 7.38x10^6 mm^4$$

9.Determine moment of inertia about the centroidal axes and radii of gyration for the T Sectionshown in figure. (16) (Nov/Dec 2009)



Solution:

$$\bar{x} = \frac{200}{2} = 100mm$$

$$\bar{y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2}$$

$$y_1 = \frac{40}{2} = 20mm$$

$$A_2 = 40 \times 120 = 4800mm^2$$

$$y_2 = 40 + \frac{120}{2} = 100mm$$

$$\bar{y} = \frac{(8000 \times 20) + (4800 \times 100)}{8000 + 4800}$$

$$\bar{y} = 50mm$$

M.I about xx-axis

$$Ixx = \left[\frac{b_1 d_1^{\ 3}}{12} + b_1 d_1 (\bar{y} - y_1)^2\right] + \left[\frac{b_2 d_2^{\ 3}}{12} + b_2 d_2 (\bar{y} - y_2)^2\right]$$

$$= \left[\frac{200 \times 40^3}{12} + 200 \times 40(50 - 20)^2\right] + \left[\frac{40 \times 120^3}{12} + 40 \times 120(50 - 100)^2\right]$$

$$Ixx = 26.03 \times 10^6 mm^4$$

M.I about yy axis

$$Iyy = \left[\frac{d_1 b_1^{\ 3}}{12}\right] + \left[\frac{d_2 b_2^{\ 3}}{12}\right]$$

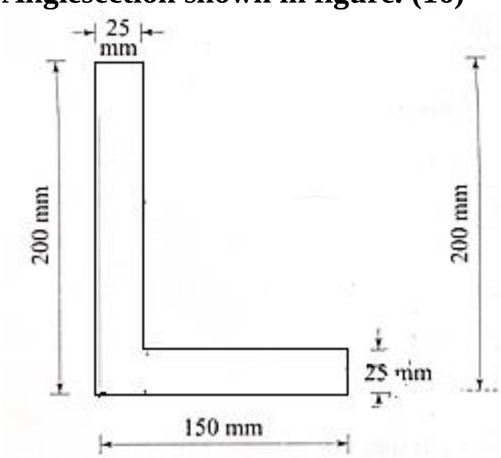
$$= \frac{40 \times 200^3}{12} + \frac{120 \times 40^3}{12} \quad Iyy = 27.31 \times 10^6 mm^4$$

Radii of gyration

$$Kx = \sqrt{\frac{Ixx}{A}} = \sqrt{\frac{23.03 \times 10^6}{12800}} \quad Kx = 45.10mm$$

$$Ky = \sqrt{\frac{Iyy}{A}} = \sqrt{\frac{27.31 \times 10^6}{12800}} \quad Ky = 46.2mm$$

10.Determine moment of inertia about the centroidal axes and radii of gyration for the Anglesection shown in figure. (16)



Solution:

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$A_1 = 200 \times 25 = 5000mm^2$$

$$x_1 = \frac{25}{2} = 12.5mm$$

$A_2 = 125 \times 125 = 3125 mm^2$

$x_2 = 25 + \frac{125}{2} = 87.5 mm$

$\bar{x} = \frac{(5000 \times 12.5) + (3125 \times 87.5)}{5000 + 3125} = 41.35 mm$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$y_1 = \frac{200}{2} = 100 mm$

$y_2 = \frac{25}{2} = 12.5 mm$

MI about xx-axis

$$\mathrm{I_{xx}} = \left[\frac{100\mathrm{x}25^3}{12} + 100\mathrm{x}25(100 - 187.5)^2 + \left[\frac{25\mathrm{x}150^3}{12}\right] + \frac{100\mathrm{x}25^3}{12} + 100\mathrm{x}25(100 - 12.5)^2\right]$$

$$= \left[\frac{25 \times 200^3}{12} + 25 \times 200(66.35 - 100)^2\right] + \left[\frac{125 \times 25^3}{12} + 125 \times 25(66.35 - 12.5)^2\right]$$

$Ixx = 31.55 \times 10^6 mm^4$

$$\mathrm{I_{yy}} = \left[\frac{\mathrm{d_1 b_1}^3}{12} + \mathrm{b_1 d_1}(\bar{\mathrm{x}} - \mathrm{x_1})^2 + \left[\frac{\mathrm{d_2 b_2}^3}{12}\right] + \frac{\mathrm{d_3 b_3}^3}{12} + \mathrm{b_3 d_3}(\bar{\mathrm{x}} - \mathrm{x_3})^2\right]$$

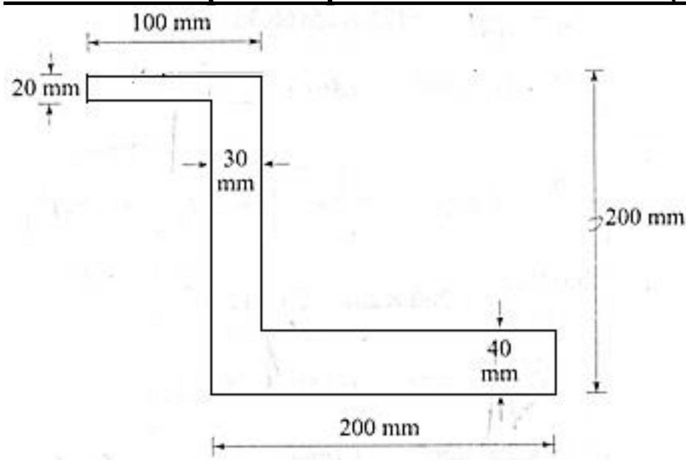
$$= \left[\frac{200 \times 25^3}{12} + 200 \times 25(41.35 - 12.5)^2\right] + \left[\frac{25 \times 125^3}{12} + 25 \times 125(41.35 - 87.5)^2\right]$$

$Iyy = 15.15 \times 10^6 mm^4$

$Kx = \sqrt{\frac{Ixx}{A}} = \sqrt{\frac{31.55 \times 10^6}{8125}} = 62.31 mm$

$Ky = \sqrt{\frac{Iyy}{A}} = \sqrt{\frac{15.15 \times 10^6}{8125}} = 43.18 mm$

11.Determine moment of inertia of the plane area shown in figure about the centroidal axes and hence compute the polar moment of inertia. (16) (Nov/Dec 2009)



Solution:

$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$

$A_1 = 100 \times 20 = 2000 mm^2$

$x_1 = \frac{100}{2} = 50 mm$

$A_2 = 140 \times 30 = 4200 mm^2$

$x_2 = 100 - 15 = 85 mm$

$A_3 = 200 \times 40 = 8000 mm^2$

$x3 = 70 + \frac{200}{2} = 170 mm$

$\bar{x} = 127.96 mm$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$

$\bar{y} = 70.56 mm$

$$Ixx = \left[\frac{b_1 d_1^{\ 3}}{12} + b_1 d_1 (\bar{y} - y_1)^2 \right] + \left[\frac{b_2 d_2^{\ 3}}{12} + b_2 d_2 (\bar{y} - y_2)^2 \right] + \left[\frac{b_3 d_3^{\ 3}}{12} + b_3 d_3 (\bar{y} - y_3)^2 \right]$$

$Ixx = 63.51 \times 10^6 mm^4$

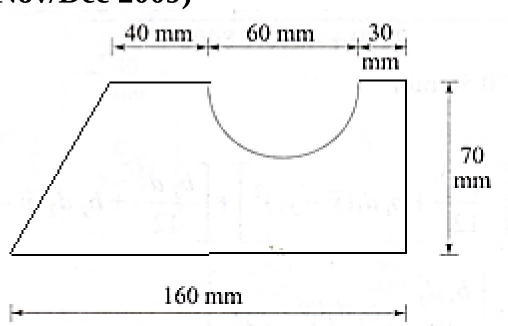
$$Iyy = \left[\frac{d_1 b_1^{\ 3}}{12} + b_1 d_1 (\bar{x} - x_1)^2 + \left[\frac{d_2 b_2^{\ 3}}{12} + b_1 d_1 (\bar{x} - x_2)^2 + \right] + \frac{d_3 b_3^{\ 3}}{12} + b_3 d_3 (\bar{x} - x_3)^2 \right]$$

$Iyy = 62.69 \times 10^6 mm^4$

$Ip = Ixx + Iyy$

$Ip = 126.20 \times 10^6 mm^4$

12.Determine moment of inertia about the centroidal axes for section shown in figure. (16) (Nov/Dec 2009)



Solution:

$\bar{x} = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$

$A_1 = 130 \times 70 = 9100 mm^2$

$x_1 = \frac{130}{2} = 65 mm$

$A_2 = \frac{1}{2} \times 30 \times 70 = 1050 mm^2$

$x_2 = 130 + (\frac{1}{3} \times 30) = 140 mm$

$A_3 = \frac{\pi d^2}{8} = \frac{\pi \times 60^2}{8} = 1414 mm^2$

$\bar{x} = 74.82mm$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3}$$

$\bar{y} = 30mm$

M.I about xx axis

$$Ixx = \left[\frac{b_1d_1^{\ 3}}{12} + b_1d_1(\bar{y} - y_1)^2\right] + \left[\frac{b_2d_2^{\ 3}}{12} + b_2d_2(\bar{y} - y_2)^2\right] - \left[0.11 \times r4 + \frac{\pi r^2}{2}(\bar{y} - y_3)^2\right]$$

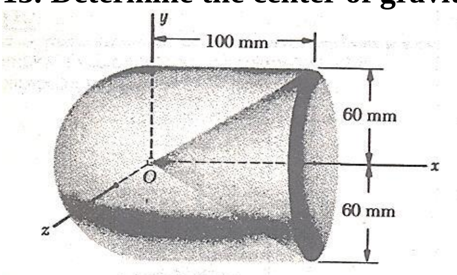
$Ixx = 3.14 \times 10^6 mm^4$

M.I about yy axis

$$Iyy = \left[\frac{d_1b_1^{\ 3}}{12} + b_1d_1(\bar{x} - x_1)^2 + \frac{d_2b_2^{\ 3}}{36} + b_2d_2(\bar{x} - x_2)^2\right] - \left[\frac{\pi r^4}{8} + \frac{\pi r^2}{2}(\bar{x} - x_3)^2\right]$$

$Iyy = 17.58 \times 10^6 mm^4$

13. Determine the center of gravity of the homogeneous body of revolution shown. (16)



Solution:

Hemisphere:

$$volume = \frac{1}{2} \times \frac{4\pi}{3} \times 60^3$$

$v = 0.452 \times 10^6 mm^3$

$\bar{x} = -22.5mm$

$\bar{x}v = -10.17 \times 10^6 mm^4$

Cylinder:

$volume = A \times 60^2 \times 100$

$v = 1.131 \times 10^6 mm^3$

$\bar{x} = 50mm$

$\bar{x}v = 56.55 \times 10^6 mm^4$

Cone:

$$volume = \frac{-\pi}{3} \times 60^2 \times 100$$

$= -0.377 \times 10^6$

$\bar{x} = 75mm$

$\bar{x}v = -28.28 \times 10^6 mm^4$

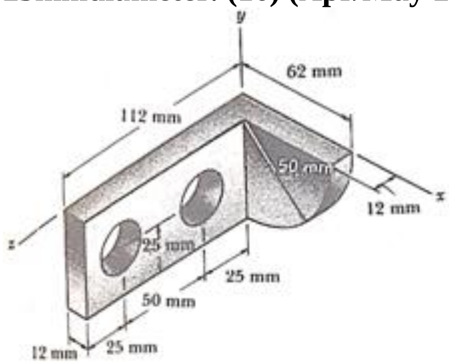
$\sum v = 1.206 \times 10^6$

$\sum \bar{x}v = 18.10 \times 10^6$

$\bar{x}\sum v = \sum \bar{x}v$

$\bar{x}(1.206\times10^6) = 18.10\times10^6 = \bar{x} = 15mm$

14. Locate the center of gravity of the steel machine element shown. Both holes are of 25mmdiameter. (16) (Apr/May 2007)



Solution:

The machine elements is seen to consist of a rectangular parallelepiped plus a quarter cylinder minus two 25mm diameter cylinder

Component	V,mm3	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\bar{x}v$	$\bar{y}v$	$\bar{z}v$
I	$112\times50\times12 = 67200$	6	-25	56	403200	-1680000	3763200
II	$\frac{1}{4}(50^2)(12) = 23562$	33.22	-21.2	6	782730	-499990	141370
III	$-\pi(12.5^2)(12) = -5890$	6	-25	37	-35340	147250	-217930
IV	$-\pi(12.5^2)(12) = -5890$	6	-25	87	-35340	147250	-512430
Total	78982				1115250	-1885490	3174210

$\bar{x}\sum v = \sum \bar{x}v; \bar{x}\times78982 = 1115250$

$\bar{x} = 14.12mm$

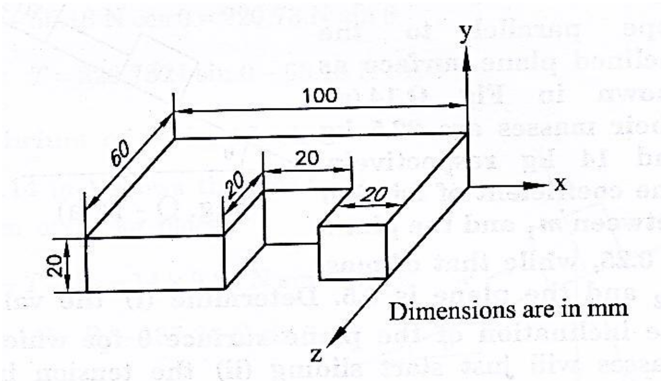
$\bar{y}\sum v = \sum \bar{y}v; \bar{y}\times78982 = -1885490$

$\bar{y} = -23.9mm$

$\bar{z}\sum v = \sum \bar{z}v; \bar{z}\times78982 = 3174210$

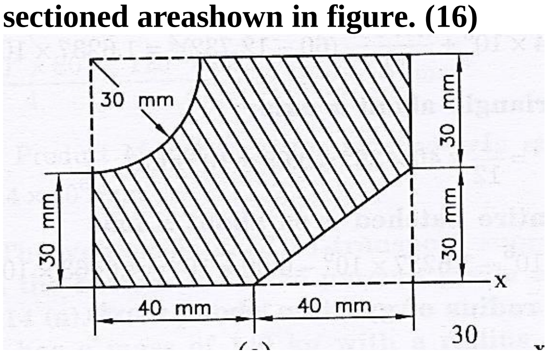
$\bar{z} = 40.2mm$

15. Find the mass moment of inertia of the rectangular block shown in figure about the vertically axis. A cuboid of 20X20X20 has been removed from the rectangular block as shown in figure. The mass of the material of the block is 7850 kg/m3. (16) (Nov-2001)



Solution:
Mass of the rectangular block
$= 0.1 \times 0.06 \times 0.02 \times 7850 = 0.942kg$
Mass MI of the block about y axis passing through its CG
$= \frac{m}{12}(100^2 + 60^2) = \frac{0.942}{12}(100^2 + 60^2)$
$= 1067.6kgmm^2$
Mass MI of the block about y axis
$= 1067.6 + 0.942 \times d^2 = 1067.6 + 0.942(30^2 + 50^2)$
$= 4270.4kgmm^2$
Mass MI of the cuboid about y axis
$= \frac{0.02 \times 0.02 \times 0.02 \times 7850}{12}(20^2 + 20^2) + 0.0628(30^2 + 50^2)$
$= 217.7kgmm^2$
Net mass MI of the block = 4270.4-217.7 =4052.7 kgmm2

16. Calculate the moment of inertia and radius of gyration about the x axis for the sectioned areashown in figure. (16)



Solution:
MI required=MI of the rectangle 1,quarter circle 2,the triangule 3
MI of the rectangle about x axis
$\frac{1}{3}bd^3 = \frac{1}{3} \times 80 \times 60^3$
$= 5.76 \times 10^6 mm^4$
MI of the quarter circle about base axis
$x^1 = Ix^1 = \frac{1}{16}xr^4 = \frac{1}{16}x \times 30^4$
Perpendicular distance of centroid from base axis
$= \frac{4r}{3\pi} = \frac{4 \times 30}{3\pi} = 12.732mm$

MI of the quarter circle about its centroidal axis

$$Ic = Ix^1 - Ad^2 = \frac{\pi}{16}\times 30^4 - \frac{1}{4}\times\pi\times 30\times 12.732^2$$

$$= 0.0444\times 10^6 mm^4$$

MI of the quarter circle about x axis

$$= 0.0444\times 10^6 + \frac{\pi}{16}\times 30^4 (60-12.732)^2$$

$$= 1.6237\times 10^6 mm^4$$

MI of the triangle about x axis

$$= \frac{1}{12}bh^3 = \frac{1}{12}\times 40\times 30^3$$

$$= 0.09\times 10^6 mm^4$$

MI of the entire hatched area about x axis

$$= 0.576\times 10^6 - 1.6237\times 10^6 - 0.09\times 10^6$$

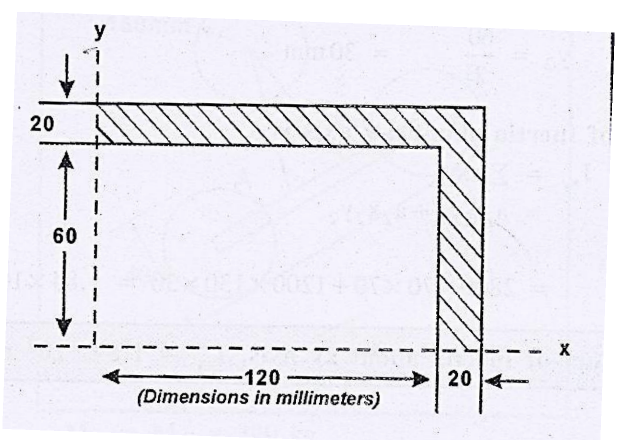
$$= 4.0463\times 10^6 mm^4$$

Radius of gyration about x axis

$$K = \sqrt{\frac{Ix}{A}} = \sqrt{\frac{4.0463\times 10^6}{3493.14}}$$

$$K = 34mm$$

17.Determine the product of inertia of the sectioned area about axes shown in figure.(16) (Jan 2003). (16)



SOLUTION:

Section 1:

A_1=a×6= 20×140=2800mm^2

X_1=140/2=70mm

Y_1=60+20/2=70mm

Section 2:

A_2=a×6= 20×60= 1200mm^2

X_2=120+20/2= 130mm

Y_2=60/2 =30mm

Product of inertia about x- axis is

I_{xy}= ∑axy

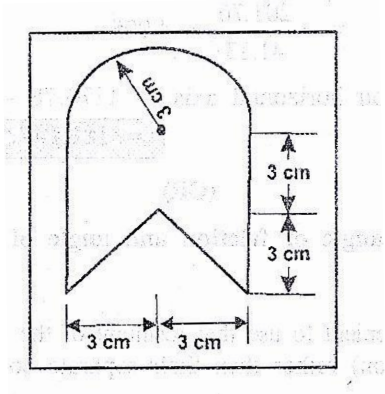
=$A_1X_1Y_1$+ $A_2X_2Y_2$

=(2800×70×70) + (1200×130×130)

=1.84× 10^{7}mm^4

18. Compute the second moment of area of the plane surface shown in figure about its

horizontalcentroidal axis. (16) (Nov/Dec 2003)



SOLUTION:

S.NO	A	Y	AY	½ BD^3	AY^2
1	36	7	108	432	
2	14.13	7.273	102.76	0.1184=8.91	743.37
3	-9	1	-9	-13.5	
	41.13		201.76	1170.78	

Y=201.76/41.13= 4.965

As one equation horizontal axis = 1170.78-201.76×4.908

= 181.1472

19. Derive the formula for the parallel axis theorem. (16)

G- Centroid of plane area

h- Distance between the two axis x-x and AB

I_{AB}- M.I. of area about AB

A- Area of section

Moment of inertia of area dA about x-x axis

$=dAy^2$

Moment of inertia of the to the area about x-x axis I_{XX}

$=\int y^2 dA$

Moment of inertia of the area dA about AB

$=dA(h+y)^2= dA\ (h^2+2yh+y^2)$

Moment of inertia of the total area A about AB

$I_{ab}=\int (h^2+ 2yh+yz)\ dA$

Where $\int h^2 dA= ab^2$

$\int y^2 dA = I_{xx}$ (or) I_a

$2h\int ydA = 0$

$I_{ab}= I_a+Ah^2$

It states that, if the moment of inertia of a plane area about an axis through its centre of gravity be denoted as I_a, then the moment of inertia of the area about axis AB parallel to the first and at a distance of h from the C.G.

20. Derive the expression for perpendicular axis theorem. (16)

Consider a strip of area dA

Let x ---- distance of dA from the axis OY

Y ---- distance of dA from the axis OX

Z ---- distance of dA from the axis OZ

$z^2 = x^2 + y^2$

Moment of inertia of area Z axis

$I_{zz} = \int dA\, z^2$

$= \int dA\, (x^2 + y^2)$

$= \int x^2 dA + \int y^2 dA$

$I_{zz} = I_{yy} + I_{xx}$

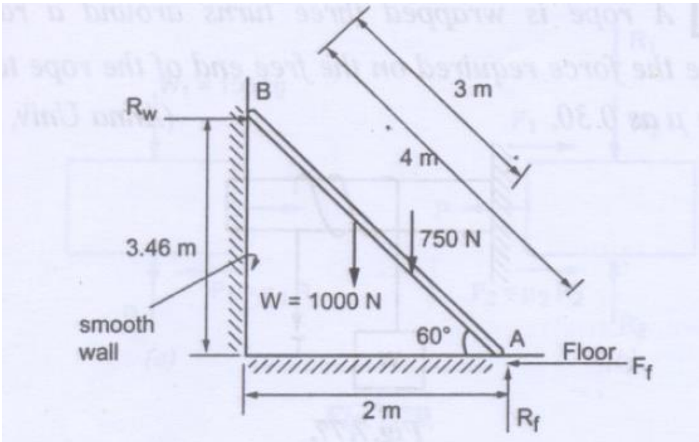
It states that moment of inertia of a plane laminar about an axis perpendicular to the laminar and passing through its centroid is equal to sum of the Moment of inertia of the lamina about two mutually perpendicular axes passing through centroid and in the plane of lamina of area A with xx, yy, zz passing through the centroid G.

Unit -4

FRICTION

Frictional force – Laws of Coloumb friction – simple contact friction – Rolling resistance – Belt friction. Translation and Rotation of Rigid Bodies – Velocity and acceleration – General Plane motion.

1. **A ladder of weight 1000 N and length 4 m rests as shown in fig. If a 750 N weight is applied at a distance of 3 m from the top of ladder, it is at the point of sliding. Determine the coefficient of friction between ladder and the floor. (Anna univ, May/June 2010) (16)**



Solution:

Let the normal reaction at the floor and the wall be Ra and Rb respectively.

Frictional force at the floor=μ Ra

Frictional force at the wall =μ RB

Where,

μ=coefficient of friction.

Resolving the forces on the ladder horizontally and vertically,

$R_B = \mu R_A$ −−−−−−−1

$R_A + \mu R_B = 1750$ −−−−−−−2

From equation 1 & 2,

$\mu(\mu R_A) + R_A = 1750$

$\mu^2 R_A + R_A = 1750$

$R_A(\mu^2 + 1) = 1750$

$R_A = 1750/(\mu^2 + 1)$

$R_B = \dfrac{1750\mu}{(\mu^2 + 1)}$

Taking moment about A,

$1000 \times (3\cos 60) + 750 \times (1 \times \cos 60) = R_B \times 3.48 \sin 60 + \mu R_B \times 3.48 \cos 60$

$(1500 + 375) = R_B(3.0137) + 1.74\mu R_B$

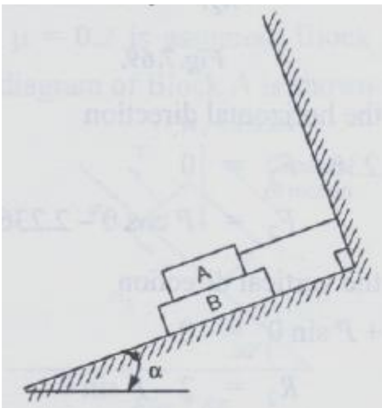
$1875 = 3.0137 R_B + 1.74\mu R_B$

$1875 = 3.0137\left[\dfrac{1750\mu}{\mu^2 + 1}\right] + \left[1.74\left[\dfrac{1750\mu}{\mu^2 + 1}\right]\right]$

$$\mu^2 + 4.5076\mu - 1.602 = 0$$

$$\mu = 0.331$$

2. **From the fig calculate α so that the motion of lower block can just slide down from the plane. The weight A and B are 30 N and 90 N respectively. The coefficient of friction for all contact surfaces is 1/3. (Anna univ, Nov/Dec 2011) (16)**



Givendata:

$W_A = 30N$, $W_B = 90N$

$$\mu = \frac{1}{3} = 0.33$$

Solution:

Considering 30N block

Resolving the forces vertically,

$$\sum \text{Fv} = 0, \quad \text{R}_1 - 30\cos\alpha = 0$$

$$\text{R}_1 = 30\cos\alpha ------1$$

From law of friction, $F_1 = \frac{1}{4}\text{R}_1$

$$F_1 = \frac{1}{3} \times 30\cos\alpha,$$

$$F_1 = 10\cos\alpha -----2$$

Considering 90N block:

Resolving the forces vertically,

$$\sum \text{Fv} = 0; \ \text{R}_2 - \text{R}_1, -90\cos\alpha = 0$$

$$\text{R}_2 = \text{R}_1 + 90\cos\alpha$$

Substituting the value of (R1) from 1 we get

$$\text{R}_2 = (30\cos\alpha + 90\cos\alpha)$$

$$\text{R}_2 = 120\cos\alpha ------3$$

$$F_2 = \frac{1}{3}\text{R}_2$$

$$F_2 = \frac{1}{3} 120 \cos\alpha - - - - - 4$$

$$F_2 = 40\cos\alpha$$

To find the angle of inclination of plane:

Resolving the forces horizontally,

$$\sum F_H = 0,$$

$$F_1 + F_2 - 90\sin\alpha = 0$$

$$(10\cos\alpha + 40\cos\alpha - 90\sin\alpha) = 0$$

$$50\cos\alpha = 90\sin\alpha$$

$$\tan\alpha = 50/90$$

$$\alpha = 29^\circ 03'.$$

3. An effort of 150 N is required to just move a body up a rough plane inclined at just 12º with the horizontal, the force being parallel to the plane. If the plane were 15º with the horizontal, the force required was 172 N. Find the weight of the body and the coefficient of friction. (Anna univ, Nov/Dec 2007) (16)

Given data:

$$P_1 = 150N; \alpha_1 = 12^\circ$$

$$P_2 = 172N; \alpha_2 = 15^\circ$$

$$W = ?; \mu = ?$$

Soiution:

For keeping the body in equilibrium, when the force parallel to the inclined plane,

The maximum force can be calculated by the following formula,

$$\mathrm{P} = \mathrm{W}(\mu\cos\alpha + \sin\alpha)$$

Case(i)

$$P_1 = 150N; \alpha_1 = 12^\circ$$

$$150 = \mathrm{W}(\mu\cos 12^\circ + \sin 12^\circ)$$

$$150 = 0.978\mu\mathrm{W} + 0.2079\mathrm{W} ------1$$

Case(ii)

$$P_2 = 172N; \alpha_2 = 15^\circ$$

$172 = W(\mu \cos 15^\circ + \sin 15^\circ)$

$172 = 0.9659\mu W + 0.2588W$ ------2

Divide equation 1 by 0.978, we get

$153.37 = \mu W + 0.2125W$

Divide eqn 2 by 0.9659 we get

$178.07 = \mu W + 0.2679W$

Solving eqn 3,4………. $W = 445.85N$

Sub the value of W in eqn 4

$178.07 = (\mu(445.85) + (0.2679 \times 445.85))$

$\mu = 0.131$.

4. **Find the least value of P for the problem shown in fig to cause the motion to impend. (Anna univ, Nov/Dec 2011) (16)**

Given data:

WA=2.5KN,

WB=2KN

α=50°,θ=50°

solution:

considering the block(B);

normal reaction=weight of the body

RB=2KN

Resolving the forces horizontally,

$$-p\cos 50^\circ + F_B = 0$$

$$\sum F_H = 0, -p\cos 50^\circ + \mu_B R_B = 0$$

Assume , coefficient of friction between block (B) and the surface, $\mu_B = 0.25$

$$-p\cos 50^\circ + 0.25R_B = 0 -----1$$

Resolving forces vertically,

$$R_B + 2 + p\sin 50^\circ = 0$$

Considering the block (A)

Resolving the forces vertically,

$$R_A = W\cos\alpha$$

$$R_A = 2.5\cos 50^\circ = 1.607KN$$

Resolving the forces horizontally,

$$P_A = F_A + W\sin\alpha$$

$$P_A = \mu_A R_A + W\sin\alpha$$

$$P_A = \mu_A(1.607) + 2\times\sin 50^\circ \qquad (\mu_A = 0.30)$$

$$P_A = 2.014KN$$

Considering the block (B):

Resolving the forces vertically,

$$\sum Fv = 0, -W + R + P\sin\theta = 0$$

$$-2 + R + p\sin 50^\circ = 0$$

$$R = (-p\sin 50^\circ + 2)$$

Resolving the forces horizontally,

$$\sum F_H = 0, \quad p\cos\theta - F + P_A = 0$$

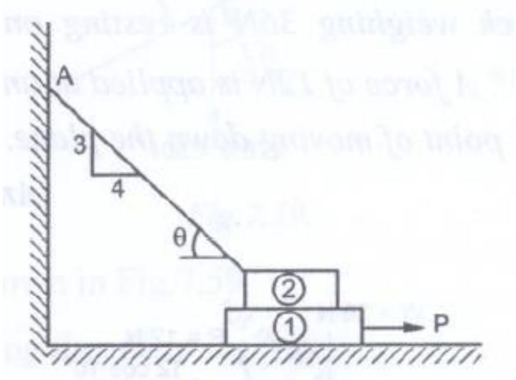
$$P\cos 50^\circ - \mu R + 2.014 = 0$$

$$P\cos 50^\circ - \mu(-p\sin 50^\circ + 2) + 2.014 = 0$$

$$p = 1.202KN.$$

5. A block of weight 1290 N rests on a horizontal surface and supports another block of weight 570 N on top of it as shown in fig. Find the force P applied to the lower block that will be necessary to cause motion to impend. The coefficient of friction between blocks 1 and 2 is 0.25 and between block 1 and the floor is 0.40.(Anna univ, May/June 2011)

(16)



Givendata:

W1=1290N; W2=570N

Let

Coefficient of friction between 1 and 2 is $\mu1=0.25$

Coefficient of friction between 1 and floor is $\mu2=0.40$

Solution:

Considering block 2

$\tan\theta = \frac{3}{4}$, $\theta = \tan^{-1}(3/4)$, $\theta = 36^{\circ}52$

Resolving the forces vertically,

$$\sum Fv = 0; (R1 + T\sin 36^{\circ}52 - 570) = 0$$

$$T\sin 36^{\circ}52 = 570 - R_1 ----1$$

Resolving the forces horizontally,

$$\sum F_H = 0; T\cos 36^{\circ}52 - F_1 = 0$$

$$T\cos 36^{\circ}52 = F_1, \quad T\cos 36^{\circ}52 = \mu R_1$$

$$T\cos 36^{\circ}52 = 0.25R_1 -----2$$

Using equation 1 & 2

$$0.7499 = \frac{570 - R_1}{0.25R_1}$$

$$0.1875R_1 = 570 - R_1, \; R_1 = 480N$$

$$F_1 = \mu R_1, = 0.25 \times 480 = 120N$$

Considering block 2

The downward force of block 2 is equal to R1 will also act along the weight of the block

Resolve the forces vertically

$$\sum Fv = 0, R_2 = 1290 + R_1$$

$$R_2 = 1290 + 480 = 1770N$$

$$F_2 = \mu_2 R_2$$

$$F2 = 0.40 \times 1770 = 708N$$

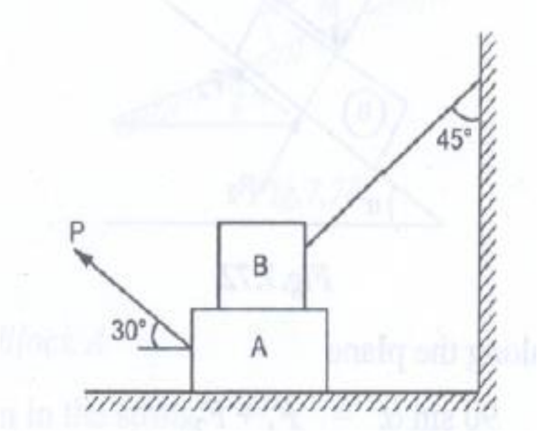
Resolve the forces horizontally

$$\sum F_H = 0, \; p = F_1 + F_2 = 120 + 708$$

$$P = 828N.$$

6. **Block A weighing 1.5 kN rests on a horizontal plane and supports another block weighing 500 N on top of it as shown in fig. The block B is attached to a vertical wall by an inclined string, which makes an angle of 45° with the vertical. What should be the value of force P acting at an angle of 30° to the horizontal to cause the motion of the lower block to impend? Take μ= 0.28 for all surfaces. (Anna univ, May/June 2011)**

(16)



Given data:

$W_A = 1.5kN; W_B = 500N$

$\mu = 0.28$

Solution:

Considering Block(B)

Resolving the forces horizontally

$\sum Fx = 0,$

$R_1 + T\sin 45^\circ - 500 = 0$

$T\sin 45^\circ = 500 - R_1 ----1$

Resolving the forces vertically

$\sum Fy = 0$

$T\cos 45^\circ - F_1 = 0$

$T\cos 45^\circ - \mu R_1 = 0$

$T\cos 45^\circ - 0.28R_1 = 0 -----2$

using equations 1&2

$$\tan 45^\circ = \frac{500 - R_1}{0.28R_1}$$

$0.28R_1 = 500 - R_1$

$1.28R_1 = 500$

$R_1 = 390.625N$

$F_1 = \mu R_1$

$F_1 = 109.375N$

Considering the block A

Resolving the forces vertically,

$$\sum Fv = 0, \quad R_2 = 1500 + R_1 - P\sin 30^\circ$$

$$F_2 = \mu R_2, \quad F_2 = 0.28(1500 + 390.625 - P\sin 30^\circ)$$

Resolving the forces horizontally,

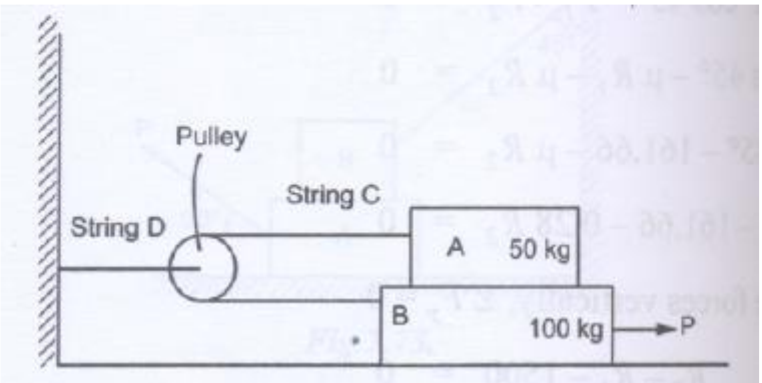
$$\sum F_H = 0$$

$$P\sin 30^\circ = F_1 + F_2$$

$$P\sin 30^\circ = 109.375 + 0.28(1890.625 - P\sin 30^\circ)$$

$$1.28\sin 30^\circ = 638.75, \quad P = 998.05N.$$

7. **Two block A and B of mass 50 kg and 100 kg respectively are connected by a string C which passes through a frictionless pulley connected with the fixed wall by another string D as shown in fig. Find the force P required to pull the block B. Also find the tension in the string D. Take coefficient of friction for all contact surfaces as 0.3. (Anna univ, Nov/Dec 2010) (16)**



Given data:

$W_A = 50kg = 490.5N$

$W_B = 100kg = 981N$

$\mu = 0.3$

Solution:

Considering Block(A)

Weight of block A is equal to the normal reaction RA

$W_A = 490.5N = R_A$

$F_A = \mu R_A$

$F_A = 0.3R_A = 0.3 \times 490.5 = 147.15N$

Since, A does not move, the tension T also,

$T = F_A = 147.15N$

Considering block(B)

Resolving the forces horizontally,

$$\sum F_H = 0$$
$$-p + \mu R_B + T = 0$$
$$-p + (0.3 \times 1471.5) - T_2 = 0$$
$$-p = -1471.5 - T_2$$
$$p = 1471.65N$$

8. A screw jack has square threads of mean diameter of 10 cm and pitch 1.25 cm. Determine the force that must be applied to the end of 50 cm lever (i) to raise (ii) to lower a weight of 50 kN. Find the efficiency of the jack. Is itself- locking? Assume μ= 0.20. (Anna univ, Nov/Dec 2002) (16)

Given Data:

Mean diameter,d=10cm $\Rightarrow$ r=5cm

lead = pitch = 1.25cm

Sol:

Length of lever (a) = 50cm

Lead angle , θ=tan$^{-1}\left[\dfrac{L}{2\pi r}\right]$

$$\theta = \tan^{-1}\left[\frac{1.25}{2\pi X 5}\right]$$
$$\theta = \tan^{-1}[0.0398]$$
$$\theta = 2.279°$$
$$\mu = \tan\phi$$
$$0.2 = \tan\phi$$
$$\phi = \tan^{-1}(0.2)$$
$$\phi = 11.31°$$

(ii) Force required to raise the weight of 50KN

$$P1 = \frac{W_r}{a}(\tan\theta + \phi)$$
$$P_1 = \frac{50x5}{50}\tan(2.27 + 11.34)$$
$$\mathrm{P}_1 = 1.189\,\mathrm{KN}$$
$$\mu = \frac{\tan\theta}{\tan(\theta + \phi)}$$
$$\mu = \frac{\tan 2.279°}{\tan(2.279° + 11.31°)}$$
$$\mu = 0.165$$

(ii) Force required to lower the weight

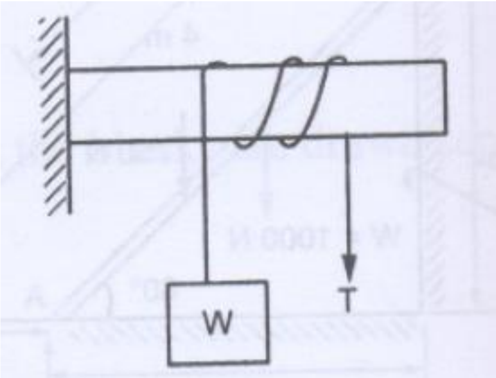
$$Q_1 = \frac{W_r}{a}\tan(\phi - \theta)$$

$$Q_1 = \frac{50X5}{50}\tan(11.30^0 - 2.279^0)$$

$$Q_1 = 0.795KN$$

IT is self locking because the friction angle (θ) is larger than the lead angle (θ) .

9. (i) A rope is wrapped three turns around a rod as shown in fig. Determine the force required on the free end of the rope to support a load of W = 20 kN, Take μ= 0.30 (Anna univ, May/June 2010) (8)



Given Data :

μ=0.30

$W = 20KN$

Solution:

Angle of Contact

$= \theta = 3$ turns

θ=3x2Π

θ=6Π

Formula :

$$\frac{T_2}{T_1} = e^{\mu\theta}$$

Where,

$T_1 =$ *Force* required on the free end of the rope

$T_2 =$ Force on the other end which is equal to W=20KN

T_2=20KN

$T_1 = {T_2}/{e^{\mu\theta}}$

$$T_1 = \frac{20}{e^{(0.3 \text{ x } 6\Pi)}}$$

$$T_1 = \frac{20}{e^{5.654}}$$

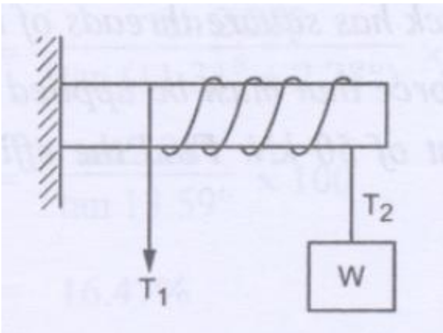
$$T_1 = \frac{20}{284.43}$$

$T_1 = 0.070069$ KN

$T_1 = 70.069$ N

The force required at the free end of the rope = 70.069 N

(ii) A rope is wrapped three and a half turns around a cylinder as shown in fig. Determine the force T_1 exerted on the free end of the rope that is required to support a 1 kN weight. The coefficient of friction between the rope and the cylinder is 0.25.
(Anna univ, Dec/Jan 2003) (8)



Sol :

$T_2 = 1$ KN

$\mu = 0.25$

$T_1 = T_2 e^{\mu\theta}$

$$\theta = 3\frac{1}{2} \text{ x } 2\pi$$

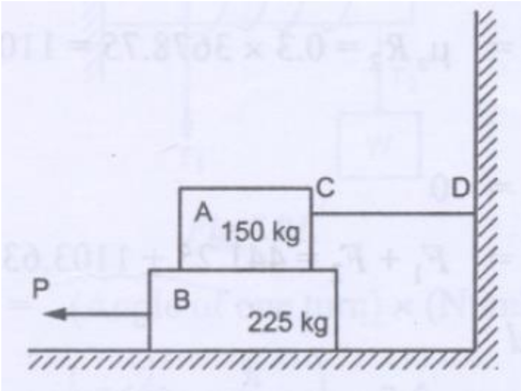
$\theta = 7\pi$

$T_1 = T_2 e^{\mu\theta}$

$T_1 = 1\text{x } e^{(0.25 \text{ x } 7\pi)}$

$T_1 = 244.15$ KN

10. **Determine the smallest force P required to move the block B shown in fig. If (i) block A is restrained by cable CD, (ii) the cable CD is removed. Take μ_s= 0.30 and μ_k= 0.25**
(Anna univ, Nov/Dec 2004) (16)



Solution:

$$\downarrow (150X9.81) = 1471.5N$$

$$0.3 \text{ x } 1471.5 \leftarrow \boxed{A} \rightarrow \text{ T = 441.45}$$

$$\uparrow 1471.5 = (150X9.81)$$

$$\downarrow 3678.75 = [(250X9.81) + (150X9.81)]$$

$$P \leftarrow \boxed{B} \rightarrow \text{ 1103.625}$$

$$\uparrow 3678.75$$

(i) Block ‘A’ is restrained by cable ‘CD’

$$P = \mu X[[(250 + 150)X9.81] + (\mu X150X9.81)]$$

$$p = (1103.625 + 441.45)$$

$$P = 1545.045N$$

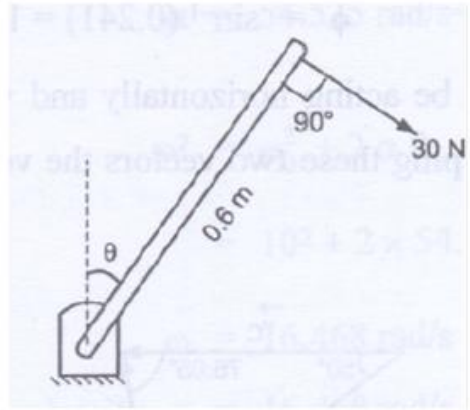
(ii) Cable ‘CD’ is resolved

$$P = \mu X[(250 + 150)X9.81]$$

$$p = [(0.30X(400X9.81)]$$

$$P = 1103.625N$$

11. The uniform 8 kg rod shown in fig is acted upon by 30 N force which always acts perpendicular to the bar. If the bar has an initial clockwise angular velocity ω= 10 rad/sec when ϴ =0°, determine its angular velocity at the instant ϴ =90°.
(Anna univ, Nov/Dec 2002) (16)



Sol:

$$I_A = \frac{ml^2}{3}$$

$$I_A = \frac{5X\,0.6^2}{3}$$

$$I_A = 0.60$$

$$M_A = (0.6a + 49.5X\,0.3\sin\theta + 30 + 0.6)$$

$$0.6a = 14.85\sin\theta + 18$$

$$\frac{d\theta^2}{dt^2} = \alpha = 24.75\sin\theta + 30$$

$$\frac{dw}{dt} = 24.75\sin\theta + 30$$

$$\frac{dw}{d\theta}W = 24.75\sin\theta$$

$$\int_{w_1}^{w_2} dwW = \int (24.75\sin\theta + 30)d\theta$$

$$\left[\frac{(w_2^{\ 2} - w_1^{\ 2})}{2} = (-24.75\cos\theta + 300)\right]_d^{\frac{\pi}{2}}$$

$$\frac{w_2^{\ 2}}{2} - \frac{100}{2} = 30X\,\frac{\pi}{2} + 24.75$$

$$\frac{w_2^{\ 2}}{2} = 15\pi + 24.75 + 50$$

$$\frac{w_2^{\ 2}}{2} = 121.873$$

$$w_2^{\ 2} = 243.74$$

$$w_2^{\ 2} = 15.612\ radi/\sec$$

Angular Velocity at ,**θ** = 90^0 is 15.612 radi / sec.

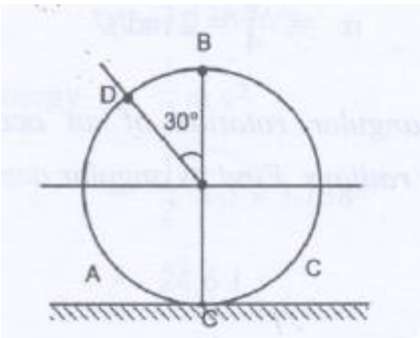
12. An automobile travels to the right at a constant speed of 72 km/hr. The diameter of wheel is 560 mm. Determine the magnitude and direction of the following:

(i) Angular velocity of the wheel

(ii) Velocity of the point B

(iii) Velocity of the point C

(iv) Velocity of the point D (Anna univ, Nov/Dec 2002) (16)



Given data:

Constant speed=72 Km/hr,=20m/sec

Diameter of the wheel= 560mm=0.56m

Solution:

The velocity at point O must be zero,(because the wheel rolls without slipping and the point O is in contact with the surface).

(i) Angular velocity of the wheel(W)

$w = r\theta$

Where , r=radius of the wheel

$r = \frac{D}{2}$, $r = \frac{0.56}{2} = 0.28m$

$w = 0.28 \times 30$, $w = 8.4 rad/\sec$.

Velocity at point 0, $V_0 = V_E - V_{o/E}$

$V_0 = 0; V_E = 20m/\sec$

Relative velocity of the point with respect to E,

$V_{o/E} = \gamma w$

(ii) Velocity at point(c):

Velocity at point(c) is , $V_c = V_E + V_{c/E}$

$$V_C = \sqrt{V_E{}^2 + V_{C/E}{}^2 + 2.V_E.V_{c/E}\cos 120}$$

$$V_C = \sqrt{202 + \gamma w2 + 2 \times 20 \times \gamma w \times \cos 120}$$

$$V_C = 18.933m/\sec$$

(iii) Velocity at point (B)

Velocity at point (B)= $V_B = V_E + V_{B/E}$

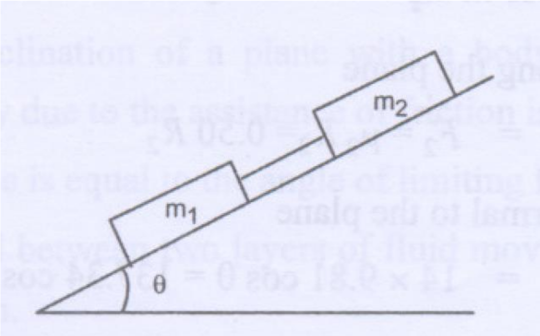
$$V_B = 20 + 8.4 = 28.4m/\sec$$

(iv) Velocity at point (D):

$$V_D = \sqrt{V_E{}^2 + V_{D/E}{}^2 + 2.V_E.V_{D/E}\cos\theta}$$

$$V_D = \sqrt{20^2 + (0.28 \times 8.4)^2 + 2 \times 20 \times 0.28 \times 8.4 \cos 30}$$
$$V_D = 22.068 m/\sec$$

13. Two masses m1 and m2 are tied together by a rope parallel to the inclined plane surface, as shown in fig. Their masses are 22.5 kg and 14 kg respectively. The coefficient of friction between m1 and the plane is 0.25, while that of mass m2 and the plane is 0.5. Determine (i) the value of the inclination of the plane surface ϴ for which the masses will just start sliding, (ii) the tension in the rope.
(Anna univ, May/June 2001) (16)



Given data:
M1=22.5kg , w1=22.5*9.81=220.725N
m2=14kg, w2=14*9.81=137.34
solution:

let, µ1=coefficient of friction between mass m1 and the plane=0.25

µ2=co efficient of friction between mass m2 and the plane=0.5

considering Block(m1)

resolving the forces along the plane,

$$-w_1 \sin\theta + 0.25R_1 + T = 0$$

$$-w_1 \sin\theta + 0.25(w_1 \cos\theta) + T = 0$$

$$T = w_1 \sin\theta - 0.25(w_1 \cos\theta) ----1$$

resolving the forces along the plane,

$$(-w_2 \sin\theta - T + 0.5R_2) = 0$$

$$T = -w_2 \sin\theta + 0.5w_2 \cos\theta ----2$$

Equating 1&2

$$w_1 \sin\theta - 0.25(w_1 \cos\theta) = -w_2 \sin\theta + 0.5w_2 \cos\theta$$

Dividing both sides by cosϴ,

$$w_1 \tan\theta - 0.25w_1 = -w_1 \tan\theta + 0.5w_2$$

$\tan\theta(w_1 + w_2) = 0.25w_1 + 0.5w_2$ =123.85

$$\tan\theta = \frac{123.85}{(220.725 + 137034)}$$

$\tan\theta = 0.3459$, $\theta = \tan^{-1}(0.3459)$, $\theta = 19.08$

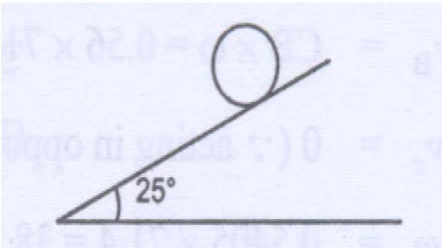
The value of the inclination of the plane surface $\theta = 19.08$ in which the masses will just start sliding.

Substituting $\theta = 19.08$ in eqn 1

$T = 220.725\sin 19.08 - 0.25(220.725)\cos 19.08$

$T = 20N$

14. A cast iron hoop of radius 200 mm is released from rest on a 25° incline as shown in fig. Find the angular acceleration of the hoop and the time taken by it to move a distance of 4 m down the slope. Take μs= 0.25. Assume that the hoop rolls without slipping. (Anna univ, Nov/Dec 2002) (16)



Given data:
R=200mm=0.2m
θ=25°
distance, d=4m,us=0.25
solution:
assume the hoop rolls is weighing at 100n.

$$M_A = \frac{100}{9.81} = 10.19$$

$r_A = 0.2m, u_s = 0.25$

W.K.T. $Fr_B = J_B\theta_B$

$$F = \frac{J_B\theta_B}{r_B}, \text{ where }, \theta_B = \frac{a_B}{\mu_B}$$

$$M_B\, g\sin\theta = M_B a_B\left[\frac{K_B}{r_B^{\,2}} + 1\right] - - - - - 1$$

Where as,

$$M_B\, g\sin\theta = \frac{J_B\theta}{r_B} + M_B a_B$$

$$M_B g \sin\theta = M_B \frac{K_B{}^2 a_B}{r_B{}^2} + M_B a_B$$

$$M_B g \sin\theta = M_B a_B \left[\frac{K_B{}^2}{r_B{}^2} + 1\right]$$

$$a_B = \frac{M_B g \sin\theta}{\left[\frac{K_B}{r_B}\right]^2 + 1}$$

$$K_B = K_S = \mu_S = 0.25$$

$$a_B = \frac{9.81 \sin 25}{\left[\frac{0.25}{0.2}\right]^2 + 1}$$

$$a_B = \frac{4.14589}{2.5625} = 1.62 m/\sec$$

a_B =angular acceleration of hoop

15. The given fig shows a stepped pulley. The smaller radius is 150 mm and the bigger radius is 200 mm. Two loads P and Q are connected by in extensible tant cords.

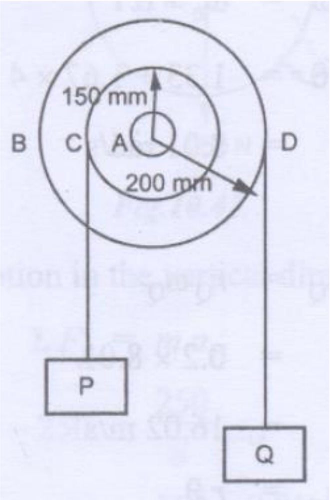
P moves with an initial velocity of 0.2 m/s and has a constant acceleration of 0.25 m/s and has a constant acceleration of 0.25 m/s² both downwards. Determine

(i) Number of revolutions turned by the pulley in 4 sec.

(ii) Velocity and the distance travelled by load Q after 4 sec.

(iii) Acceleration of point B located on the rim of the pulley at t=0. Give both magnitude and direction.

(Anna univ, Nov/Dec 2010) (16)



Given data:

$$Rp = 150 mm = 0.15 m$$

$$R_Q = 200 mm = 0.20 m$$

Initial velocity, $Wo = 0.2m/\sec$

$Acceleration = 0.25m/\sec^2$

Solution:

We know that,

$$a_p = r_p.\alpha_p$$

Where, a_p =angular acceleration at Block P

R=radius

$$\alpha_p = \frac{a_p}{r_p} = \frac{0.25}{150\times10^{-3}}, \alpha_p = 1.67 rad/\sec^2$$

Number of rotation turned by the pully t=4sec.

Angle turned, $\theta_p = W_o t + \frac{1}{2}\alpha t^2$

$$\theta_p = (0.2\times4) + (\frac{1}{2}\times1.67\times4^2)$$

$$\theta_p = 14.16 rad.$$

No of revolution, $n = \frac{\theta}{2\pi}$

$$n = \frac{14.16}{2\pi} = 2.25$$

Average velocity and distance travelled by load Q after 4 sec.

T=4sec

At load Q is turned after 4 sec, w=0

Angular velocity , $W = W_o + \alpha t$

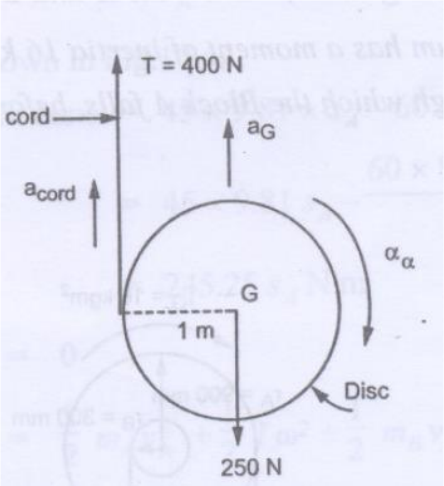
W.K.T . $\omega = \frac{2\pi N}{60}$

$$\omega = \frac{2\pi\times1.59\times10^3}{60} = 166.5 rad/\sec$$

$$W = W_o + \alpha t, \qquad 166.5 = 0 + \alpha\times4$$

$$\alpha = \frac{166.5}{4} = 41.6 rad/\sec.$$

16. A cord is wrapped on a 2 m diameter disc, which weighs 250 N. If the cord is pulled upwards with a force of 400 N, determine the acceleration of centre of gravity of the disc, the angular acceleration of the disc and the acceleration of the cord.
(Anna univ, Nov/Dec 2011) (16)



Given data:

D=2m, r=1m

Weight, w=mg=250N

Force, F=400N

Solution:

Let, T= tension in chord

Mg=weight of the cylinder

A=angular acceleration of the cylinder in (rad/sec2)

According to Newtons law,

D'Alembert's principle

No force acting in x-direction

$$\sum Fx = \max , \quad \sum Fy = ma$$

$$\sum mg = IG.d$$

$$TR = IG.d , \quad TR = \frac{mR^2}{2}.\alpha -----2$$

Solving eqn 1 & 2we get,

$$(mg - ma)R = \frac{mR^2}{2}.\alpha$$

w.k.t $\alpha = \frac{a}{R}$ sub in above eqn

$$(mg - ma)R = \frac{mR^2}{2}(\frac{a}{R})$$

$$(mg - ma) = \frac{ma}{2}$$

$$m(g - a) = \frac{ma}{2}$$

$$g - a = \frac{a}{2}, g = \frac{3}{2}a$$

$$a = \frac{2}{3}g = \frac{2}{3} \times 9.81, \qquad a = 6.54 m/s$$

Angular acceleration , $\alpha = \frac{a}{R} = \frac{6.54}{1} = 6.54 rad/\sec$

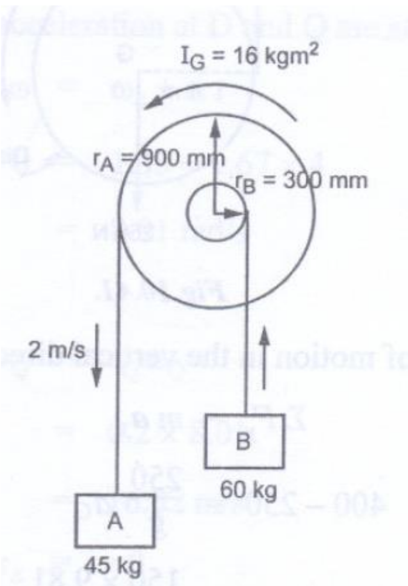
Equation 1 becomes,

$$mg - T = m.ay$$

$$(250 - 400) = m \times 6.54 = m = -22.94$$

$$C.G = I_{\alpha} = \frac{MR^2}{2} = \frac{22.94 \times 1^2}{2} = 11.47$$

17. In the fig given below, the blocks A and B have masses of 45 kg and 60 kg respectively. The drum has a moment of inertia of 16 kg/m^2 about its Axis of rotation. Find the distance through which the block A falls, before it reaches a speed of 2 m/s.
(Anna univ, May/June 2001) (16)



Given data:

W1=45kg=(45*9.81)=441.45N

W2=60kg=588.6N

M.I=16 kg/m2=156.96N/m2

Solution:

$$T_1 + 441.5a = 441.45 - - - - - 1$$

$$a = 0.9\theta; a_B = 0.3\theta = \frac{a}{3}$$

$$T_2 = 588.6 + 60\frac{a}{3} - - - - - 2$$

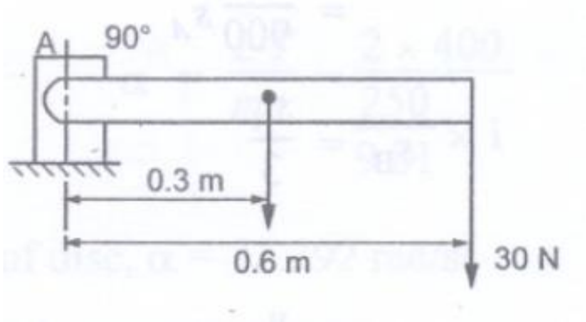
$$3\theta = T_1 \times 0.9 - T_2 \times 0.3$$
$$16\theta = 0.9T_1 - 0.3T_2$$
$$16\theta = 0.9(441.45 - 45a) - 0.3(588.6 + 20a)$$

$$16\theta = 397.3 - 40.5a - 176.58 - 6a$$
$$16\theta = 220.72 - 46.5a$$
$$(16\theta + 46.5 \times 0.9\theta) = 220.72$$

$$57.85\theta = 220.72$$
$$\theta = 3.81 rad/\sec^2$$

$$a = (0.9 \times 3.81)$$
$$a = 3.429 m/\sec^2$$
$$V_f^2 - V_i^2 = 2as$$
$$4 = (2 \times 3.429 \times s)$$
$$s = 0.583m$$

18. **A uniform 5 kg rod is shown in fig below is acted upon by 30 N force which always acts perpendicular to the bar. If the bar has an initial clockwise angular velocity, ω0= 10 rad/sec, when Θ =0, determine the angular velocity at the instant Θ =90°. (Anna univ, Nov/Dec 2011) (16)**



Given data:

W=5kg = (5*9.81)=49.05

Force, P=30N

ω°=10 rad/sec

θ=0

solution:

Angular velocity,

$$\omega = \omega\circ + \alpha T --------1$$

Initial angular velocity, $\omega\circ$=10 rad/sec

Since, $\omega = \dfrac{2\pi N}{60}$

$$\omega = r\theta$$

$$\omega = 0.3 \times 90^\circ$$

$$\omega = 27 rad.\sec$$

$$27 = 10 + \alpha T$$

$$\alpha T = 17.$$

19. A block weighing 36 N is resisting on a rough inclined plane having an inclination of 30º. A force of 12 N is applied at an angle of 10º up the plane and the block is just on the point of moving down the plane. Determine the coefficient of friction. (Anna univ, May/June 2012) (16)

Given data:

Weight of block,W=36N

Inclination angle, α =30

External force, p=12N

Force angle, θ=10

Co efficient of friction,μ=?

Solution:

The block is just on the point of moving down the plane

$$p = \frac{W(\sin\alpha - \mu\cos\alpha)}{(\cos\theta - \mu\sin\theta)}$$

$$12 = \frac{36(\sin 30^\circ - \mu\cos 30^\circ)}{(\cos 10^\circ - \mu\sin 10^\circ)}$$

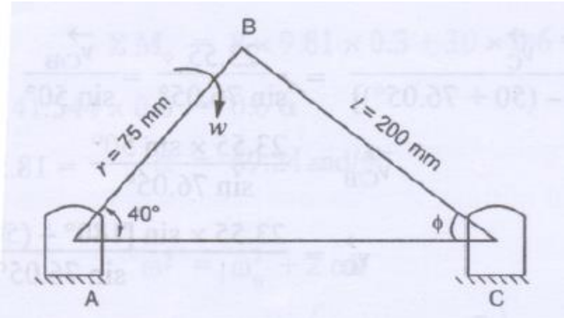
$$12(0.9849 - 0.1736\mu) = (21.1608 - 31.176\mu)$$

$$29.176\mu = 9.342$$

$$\mu = \frac{9.342}{29.176}$$

Coefficient of friction, $\mu = 0.3202$.

20. In the engine system shown in fig, the crank AB has a constant angular velocity of 3000 rpm. For the crank positioned indicated, find (i) the angular velocity of the connecting rod, (ii) velocity of piston.(Anna univ, Nov/Dec 2002) (16)



Given data:
Constant angular velocity,N=3000 rpm,
Solution:
Motion of crank,
Angular velocity,

$$\omega = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi \times 3000}{60} = 314 rad/\sec$$

The crank rotates about point A

$$V_B = \omega_{ab} \times r$$
$$= 314 \times 0.075$$
$$= 23.55 m/\sec$$

Motion of connecting rod
From triangle ABC,

$$75\sin 40 = 200\sin\phi$$
$$\phi = 13.9$$

From the vector diagram,

$$\frac{Vc}{\sin(180-(50+76.1)} = \frac{23.55}{\sin 76.1} = \frac{V_{c/B}}{\sin 50}$$

$$V_{c/B} = \frac{23.55 \times \sin 50}{\sin 76.1}$$

$$= \frac{23.55 \times 0.76}{0.9707}$$
$$= 18.58 m/\sec$$

$$Vc = \frac{23.55 \sin(180-(50+76.1)}{\sin 76.1}$$

$$= \frac{19.028}{0.9707}$$
$$Vc = 19.60 m/\sec$$

$$0.2 \times \omega_{BC} = V_{c/B}$$
$$0.2 \times \omega_{BC} = 18.58$$
$$\omega_{BC} = 92.9 rad/\sec$$

Unit -5
DYNAMICS OF PARTICLES
PART- A (2Marks)

1.Write short notes on dynamics.

Dynamics is a branch of physics (specifically classical mechanics) concerned with the study of forces and torques and their effect on motion, as opposed to kinematics, which studies the motion of objects without reference to its causes. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion. Newton established the fundamental physical laws which govern dynamics in physics. By studying his system of mechanics, dynamics can be understood.

2.Define kinematics.

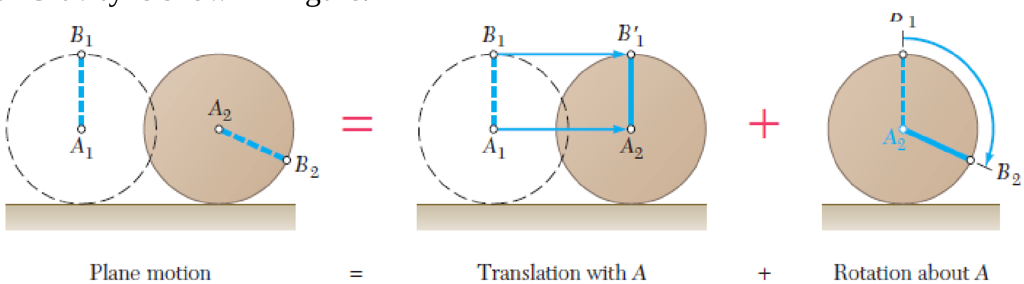
Kinematics is the branch of classical mechanics which describes the motion of points, bodies and systems of bodies without consideration of the causes of motion. To describe motion, kinematics studies the trajectories of points, lines and other geometric objects and their differential properties such as velocity and acceleration. Kinematics is used in astrophysics to describe the motion of celestial bodies and systems, and inmechanical engineering, robotics and biomechanics

3.Differentiate kinetics & kinematics. (Nov-11)

Kinetics and kinematics are two words in the study of motion and the forces that are involved in these motions that confuse a lot of people. The situation becomes confusing because these two words are similar sounding, and also because both of them are involved in the study of motion. However, while kinetics study the motion and the forces that are underlying this motion, kinematics is solely focused on the study of motion and does not take into account any forces that may be acting upon the body in motion.

4.What is general plane motion? (Nov/Dec-2012)

If a rigid body moves with both translational and rotational motion, it is said to be in general plane motion. General Plane Motion Force and Acceleration Diagrams around
the Center of Gravity is shown in figure.



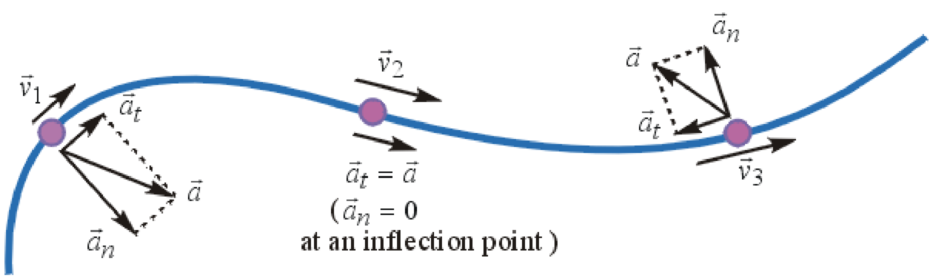
5. Define plane motion.

The motion of a rigid body such that all its points move parallel to some fixed plane. The study of plane motion reduces to the study of the motion of an invariant plane figure within its plane. Such motion is composed of translational motion together with an arbitrarily selected pole around which rotational motion occurs. Plane motion can also be represented as a series of elementary rotations about instantaneous centers of rotation that continuously change position.

6. What is rectilinear motion?(Nov/Dec-2011)

Linear motion (also called rectilinear motion) is a motion along a straight line, and can therefore be described mathematically using only one spatial dimension. The linear motion can be of two types: uniform linear motion with constant velocity or zero acceleration; non uniform linear motion with variable velocity or non-zero acceleration. The motion of a particle (a point-like object) along a line can be described by its position x, which varies with t (time). An example of linear motion is an athlete running 100m along a straight track.

1. Write about curvilinear motion.



i.Velocity, v, is always tangent to the path. See velocities , v1, v2, and v3 in the figure above. Notice that the lengths (speed) of these velocity vectors vary (v3 > v2 > v1)due to the tangential acceleration.

ii.In general, the total acceleration a is not tangent to the path; it acts toward the concave side.

iii.The tangential acceleration, at, like velocity, always acts tangent to the curve. It changes the length of the velocity vector. It may act in the opposite direction of v, slowing the particle along the path (as it does at position 3), or in the direction of v, increasing the particle's speed along the path (in positions 1 and 2).

iv.The normal acceleration, an, always acts toward the concave side of the path. Normal acceleration changes the direction of the velocity vector.

2. Define Velocity.

Velocity- Velocity is the rate of change of the displacement, the difference between the final and initial position of an object. Velocity is equivalent to a specification of its speed and direction of motion, e.g. 60 km/h to the north. Velocity is an important concept in kinematics, the branch of classical mechanics which describes the motion of bodies. Velocity is a vector physical quantity; both magnitude and direction.

3. Define acceleration.

Acceleration- Acceleration, in physics, is the rate of change of velocity of an object. An object's acceleration is the net result of any and all forces acting on the object, as described by Newton's Second Law. The SI unit for acceleration is the meter per second squared (m/s^2). Accelerations are vector quantities ,they have magnitude and direction. As a vector, the calculated net force is equal to the product of the object's mass a scalar quantity and the acceleration.

4. Write the mathematical expression for velocity.

Velocity Formula

Velocity is the measure of the speed of the object in a specific direction. It is denoted by V and is given as

$$V = \frac{\text{Displacement}}{\text{Time taken}} = \frac{S}{t}$$

Where S is the displacement and, t is the time taken. Since displacement is expressed in meters and time taken in seconds. Velocity is expressed in **meters/second** or **m/s**.

In any problem if any of these two quantities are given we can find the missing quantity using this formula.

5. Write short notes on speed vs velocity.

Speed only gives you a number that tells you how fast you are going. Velocity, because it adds direction, tells you how fast you are changing your position. Because of this difference, if your position doesn't change even if you are moving very fast, your velocity will be zero. If you were to run in place very fast, your speed may be 6 mph, but your velocity would be 0 because you aren't going anywhere. If you were to run backwards and forwards, always returning to your same spot, your velocity would be 0 again because you didn't go anywhere.

6. Write the equation of initial, final velocity formula.

Generally the initial velocity is denoted by 'u' and the final velocity is denoted by 'v.'In one dimensional motion, it has more importance to solve the displacement(s), velocity (u or v) and acceleration (a).

The following equations can be used when the particle moving with a constant acceleration.

1. v = u + at
2. s = 12(u + v)t
3. s = ut + 12a*t*2
4. *v*2 = *u*2 + 2as

Where **u** is the initial velocity
v is the final velocity
s is the displacement
a is the acceleration
t is the time taken

7. What is average velocity?

Average velocity- Velocity is defined as the rate of change of position with respect to time. Sometimes it is easier, or even necessary, to work with the *average velocity* of an object, that is to say the constant velocity, that would provide the same resultant displacement as a variable velocity, **v**(*t*), over some time period Δ*t*. Average velocity can be calculated as:

$$\bar{\boldsymbol{v}} = \frac{\Delta \boldsymbol{x}}{\Delta t}$$

The average velocity is always less than or equal to the average speed of an object. This can be seen by realizing that while distance is always strictly increasing, displacement can increase or decrease in magnitude as well as change direction.

8. Define instantaneous velocity?

Instantaneous velocity is the velocity of an object in motion at a specific point in time. This is determined similarly to average velocity, but we narrow the period of time so that it approaches zero. If an object has a standard velocity over a period of time, its average and **instantaneous velocities** may be the same. The formula for **instantaneous velocity** is *the limit as t approaches zero of the change in d over the change in t.*

9. Write the formula to find out the instantaneous velocity?

Instantaneous Velocity Formula is used to determine the instantaneous velocity of the given body at any particular instant. It is given as:

$$Instantaneous\ Velocity = Lim_{\Delta t -> 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Where x is the given function with respect to time t. The Instantaneous Velocity is expressed in m/s. If any problem contains the function of the form f(x), the instantaneous velocity is determined using the above formula.

10. Find the Instantaneous Velocity of a particle moving along a straight line with a function $x = 5t^2 + 2t + 3$ at time t = 3s?

Solution:

Given: The function is $x = 5t^2 + 2t + 3$

Differentiating the given function with respect to t, we get Instantaneous Velocity

V_{inst} = *dxdt*

= *d*(5*t*2+2*t*+3)*dt*

= 10t + 2

For time t = 3s, the Instantaneous Velocity is V(t) = 10t + 2

V(3) = 10(3) + 2 = 32m/s

Instantaneous Velocity for the given function is 32m/s.

11. Give the relationship with velocity and acceleration.

Although velocity is defined as the rate of change of position, it is often common to start with an expression for an object's acceleration. As seen by the three green tangent lines in the figure, an object's instantaneous acceleration at a point in time is the slope of the line tangent to the curve of a (***v*** vs. *t* graph at that point. In other words, acceleration is defined as the derivative of velocity with respect to time:

$$a = \frac{dv}{dt}$$

From there, we can obtain an expression for velocity as the area under an acceleration vs. time (*a* vs. *t*) graph. As above, this is done using the concept of the integral:

$$v = \int a \, dt$$

12. A steel ball is vertically thrown upwards from the top of the building 25 m above the ground with an initial velocity of 18 m/sec. Find the max height reached by the ball from the ground. (Apr/May-2003)

a. Solution:

1. V = u + at
2. 0 = 18 – 9.81 x t
3. t = 1.834
4. s = 18t – (9.81/2) t^2
5. = 16.51
6. H= 25 + s
7. H = 25 + 16.51 = 41.51 m.

13. What is the relative motion of the first body with respect to second body?

Relative motion - the motion of a point or a body with respect to a moving frame of reference that travels in a certain manner relative to some other, primary frame of reference arbitrarily called fixed. The velocity of a point in relative motion is called the relative velocity and the point's acceleration is referred to as the relative acceleration . The motion of all points of the moving frame of reference with respect to the fixed frame is in this case called vehicle motion, and the velocity and acceleration of the point of the moving system through which the point in motion passes at a given moment of time are called the vehicle velocity v_{veh} and the vehicle acceleration , respectively.

14. What is angular velocity?

The angular velocity is defined as the rate of change of angular displacement and is a vector quantity (more precisely, a pseudovector) which specifies the angular speed (rotational speed) of an object and the axis about which the object is rotating. The SI unit of angular velocity is radians per second, although it may be measured in other units such as degrees per second, degrees per hour, etc. Angular velocity is usually represented by the symbol omega (**ω**, rarely **Ω**).

The direction of the angular velocity vector is perpendicular to the plane of rotation, in a direction which is usually specified by the right-hand rule.

15. Write the equation for **Angular Velocity Formula** is given by

The rate of change of angular displacement of the particle in a given time is called angular velocity.

It is expressed as

$$\omega = \frac{d\theta}{dt}$$

Where d θ is change in angular displacement, dt is change in time t.

16. Calculate the angular velocity of the particle moving along the straight line given by $\theta = 3t^3 + 6t + 2$ when t = 5s.

Solution:

Given: $\theta = 3t^3 + 6t + 2$, time $t = 5$ s

The angular velocity is given by

$\omega = d\theta dt = 9t^2 + 6$

For $t = 5s$,

the angular velocity is

$\omega = 9(5)^2 + 6 = 175 + 6 = 181$ units/sec.

17. Find the angular velocity of a second hand of a clock?

Solution:

The second hand of the clock completes 1 complete rotation in 60 s.

Angular velocity $\omega = 2\pi$,

Time taken $t = 60$ s

The angular velocity is given by $\omega = \theta t$

$= 2\pi 60$

$= 0.1047$ rad/s.

18. What is relation b/w angular velocity and linear velocity?

Sl.No	Linear Velocity	Angular Velocity
1	It is the velocity of object travelling in the **straight line**.	It is the velocity of object traveling in the **circular or curved or angular path.**
2	It is measured in **m/sec.**	It is measured in **rad/sec.**
3	The equation of Linear velocity is: $v = xt$ **m/sec**	The equation of Angular velocity is: $\omega = \theta t$ **radians/sec**
4	The linear velocity could be constant since the object could be travelling with constant speed without changing its direction.	The angular velocity could not be constant since the body is constantly changing its direction.

19. Define linear velocity.

Linear velocity is the velocity of the object travelling in a straight line or in other words the body is said to be moving with linear velocity when its direction is not changing.

Linear velocity, as we know, depends on the distance an object travels with respect to time. The linear equation or the linear formula is given below

$v = xt$

where,

v = linear velocity

x = distance covered

t = time taken to cover distance x

20. What is angular acceleration?

Angular acceleration- the angular velocity is defined as the rate of change of angular displacement and is a vector quantity which specifies the angular speed of an object and the axis about which the object is rotating. The SI unit of angular velocity is radians per second, although it may be measured in other units such as degrees per second, degrees per hour, etc. Angular velocity is usually represented by the symbol omega (ω, rarely Ω).The direction of the angular velocity vector is perpendicular to the plane of rotation, in a direction which is usually specified by the right-hand rule.

21. What is the relative motion?

The laws of physics which apply when you are at rest on the earth also apply when you are in any reference frame which is moving at a constant velocity with respect to the earth. For example, you can toss and catch a ball in a moving bus if the motion is in a straight line at constant speed. Relative motion is the calculation of the motion of an object with regard to some other moving object. Thus, the motion is not calculated with reference to the earth, but is the velocity of the object in reference to the other moving object as if it were in a static state.

22. What are motion curves?

A motion curve is just one of the animated parameters, considered as a function of time. For example, if p1(t) is indeed our function mapping from time to the x-coordinate of the camera in a scene, we can plot it to see it as a curve. If we felt something was wrong with the camera motion, looking at that plot could instantly clue us in to moments where it might be too jerky
or overly smooth.

23. A ball dropped from a height of 1.6 m on a floor rebounds to a height of 0.9 m find the coefficient of restitution. (Apr/May-2004)

Solution:

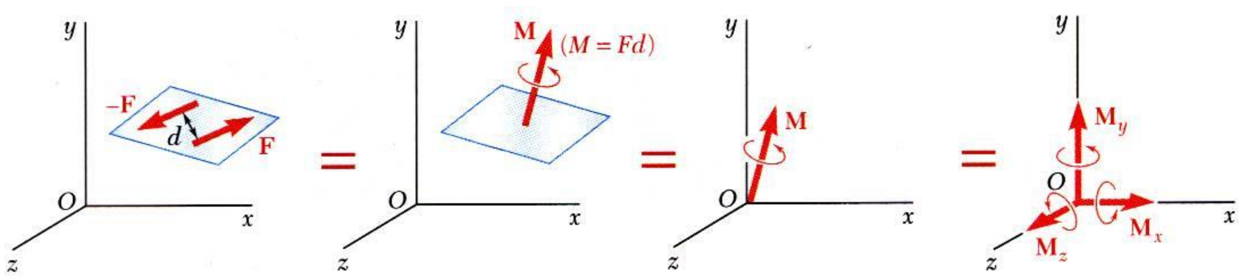
1. $V_1 = \sqrt{(2g1.6)}$
2. $V_2 = \sqrt{(2g0.9)}$

ii. $(V_2/V_1) = e$

iii. $(V_2/V_1) = 0.75$

24. What is the equation for addition of couple?

Addition couple Equation - By applying Varignon's Theorem to the Forces in the Couple, it can be proven that couples can be added and resolved as Vectors.



25. Define weight.

Weight- In science and engineering, the weight of an object is usually taken to be the force on the object due to gravity.[1][2] Its magnitude (a scalar quantity), often denoted by an italic letter W, is the product of the mass m of the object and the magnitude of the local gravitational acceleration g;[3] thus: W = mg. The unit of measurement for weight is that of force, which in the International System of Units (SI) is the newton. For example, an object with a mass of one kilogram has a weight of about 9.8 newtons on the surface of the Earth, and about one-sixth as much on the Moon.

26. Define mass.

Mass- The amount of matter in certain types of samples can be exactly determined through electrodeposition[clarification needed] or other precise processes. The mass of an exact sample is determined in part by the number and type of atoms or molecules it contains, and in part by the energy involved in binding it together Inertial mass is a measure of an object's resistance to changing its state of motion when a force is applied. It is determined by applying a force to an object and measuring the acceleration that results from that force.

27. What is projectile?

Projectile is a body thrown with an initial velocity in the vertical plane and then it moves in two dimensions under the action of gravity alone without being propelled by any engine or fuel. Its motion is called projectile motion. The path of a projectile is called its trajectory.

Examples:

A packet released from an airplane in flight.

A golf ball in flight.

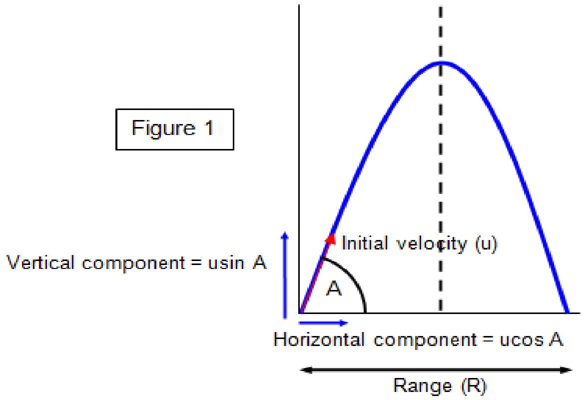
A bullet fired from a rifle.

A jet of water from a hole near the bottom of a water tank.

28. Define projectile motion.

Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown near the earth's surface, and it moves along a curved path under the action of gravity only. The only force of significance that acts on the object is gravity, which acts downward to cause a downward acceleration. In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

29. Draw velocity of projection and angle of projection of a projectile?



30. Define time of flight.

Time of flight (TOF) describes a variety of methods that measure the time that it takes for an object, particle or acoustic, electromagnetic or other wave to travel a distance through a medium. This measurement can be used for a time standard (such as an atomic fountain), as a way to measure velocity or path length through a given medium, or as a way to learn about the particle or medium (such as composition or flow rate). The traveling object may be detected directly (e.g., ion detector in mass spectrometry) or indirectly (e.g., light scattered from an object in laser doppler velocimetry).

31. Define horizontal range.

It is the horizontal distance covered during the time of flight T. Suppose a projectile is thrown from the ground level, then the range is the distance between the launch point and the landing point, where the projectile hits the ground. When the projectile comes back to the ground, the vertical displacement is zero.

A body can be projected in two ways :

Horizontal projection-When the body is given an initial velocity in the horizontal direction only.

Angular projection-When the body is thrown with an initial velocity at an angle to the horizontal direction.

32. Define angular momentum. (Nov-2003)

Angular momentum-, **moment of momentum**, **or rotational momentum** is a measure of the amount of rotation an object has, taking into account its mass, shape and speed. It is a vector quantity that represents the product of a body's rotational inertia and rotational velocity about a particular axis. The angular momentum of a system of particles (e.g. a rigid body) is the sum of angular momenta of the individual particles. For a rigid body the angular momentum can be expressed as the product of the body's moment of inertia, *I*, (i.e., a measure of an object's resistance to changes in its rotation velocity) and its angular velocity, ω.

$$\mathbf{L} = I\omega$$

33. What is law of conservation of energy?

Law of conservation of energy- the **law of conservation of energy** states that the total energy of an isolated system remains constant—it is said to be *conserved* over time. Energy can be neither created

nor be destroyed, but it can change form, for instance chemical energy can be converted to kinetic energy in the explosion of a stick of dynamite.
A consequence of the law of conservation of energy is that a perpetual motion machine of the first kind cannot exist. That is to say, no system without an external energy supply can deliver an unlimited amount of energy to its surroundings

34. What is meant by momentum?

linear momentum or **translational momentum** (pl. momenta; SI unit kg m/s, or equivalently, N s) is the product of the mass and velocity of an object. linear momentum is a vector quantity, possessing a direction as well as a magnitude:

$$\mathbf{p} = m\mathbf{v}.$$

Linear momentum is also a *conserved* quantity, meaning that if a closed system is not affected by external forces, its total linear momentum cannot change. In classical mechanics, conservation of linear momentum is implied by Newton's laws

35. What is meant by impulse?

Impulse- impulse (symbolized by **J** or **Imp**) is the integral of a force, F, over the time interval, t, for which it acts. Since force is a vector quantity, impulse is also a vector in the same direction. Impulse applied to an object produces an equivalent vector change in its linear momentum, also in the same direction. The SI unit of impulse is the newton-second (N·s), and the dimensionally equivalent unit of momentum is the kilogram-meter per second (kg·m/s). The corresponding English engineering units are the pound-second (lbf·s) and the slug-foot per second (slug·ft/s).

36. Write the equation to find out the Impulse.

An impulse may also be regarded as the change in momentum of an object to which a resultant force is applied. The impulse may be expressed in a simpler form when the mass is constant:

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}\, dt = \Delta\mathbf{p} = m\mathbf{v_2} - m\mathbf{v_1}$$

where

F is the resultant force applied,

t_1 and t_2 are times when the impulse begins and ends, respectively,

m is the mass of the object,

$\mathbf{v}_2$ is the final velocity of the object at the end of the time interval, and

$\mathbf{v}_1$ is the initial velocity of the object when the time interval begins.

37. Write short notes on potential Energy.

An object can store energy as the result of its position. For example, the heavy ball of a demolition machine is storing energy when it is held at an elevated position. This stored energy of position is referred to as potential energy. Similarly, a drawn bow is able to store energy as the result of its position. When assuming its *usual position* (i.e., when not drawn), there is no energy stored in the bow. Yet when its position is altered from its usual equilibrium position, the bow is able to store energy by virtue of its position. This stored energy of position is referred to as potential energy. **Potential energy** is the stored energy of position possessed by an object.

38. Write the Pappus and Guldinius theorems.(Apr/May12)

Theorem I:

The area of a surface of revolution is equal to the product of length of the generating curve and the distance travelled by the centroid of the curve while the surface is being generated.

Theorem II:

The volume of a body or revolution is equal to the product of the generating area and the distance travelled by the centroid of the area while the body is being generated.

39. Define principal moment of inertia.

The axis about which moments of inertia are maximum and minimum are known as principal axes. When these two axes are passing through centroid of area it is known as centroidal principal axis. Now the maximum and minimum moments of inertia are called principal moments of inertia. One of the major interest in the moment of inertia of area is determining the orientation of the orthogonal axes passing a pole on the area with maximum or minimum moment of inertia about the axes.

40. Write short notes on Kinetic Energy.

Kinetic energy is the energy of motion. An object that has motion - whether it is vertical or horizontal motion - has kinetic energy. There are many forms of kinetic energy - vibrational (the energy due to vibrational motion), rotational (the energy due to rotational motion), and translational (the energy due to motion from one location to another). To keep matters simple, we will focus upon translational kinetic energy. The amount of translational kinetic energy (from here on, the phrase kinetic energy will refer to translational kinetic energy) that an object has depends upon two variables: the mass (m) of the object and the speed (v) of the object. The following equation is used to represent the kinetic energy (KE) of an object.

$$\mathbf{KE = 0.5 \cdot m \cdot v^2}$$

41. Write short notes on Mechanical Energy and elastic potential energy.

Energy stored in food or fuel that is transformed into work. In the process of doing work, the object that is doing the work exchanges energy with the object upon which the work is done. When the work is done upon the object, that object gains energy. The energy acquired by the objects upon which work is done is known as **mechanical energy**. Mechanical energy is the energy that is possessed by an object due to its motion or due to its position. Mechanical energy can be either kinetic energy (energy of motion) or potential energy (stored energy of position).

42. Write short notes on power.

The expression for power is work/time. And since the expression for work is force x displacement, the expression for power can be rewritten as (force*displacement)/time. Since the expression for velocity is displacement/time, the expression for power can be rewritten once more as force*velocity. This is shown below.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \cdot \text{Displacement}}{\text{Time}}$$

$$\text{Power} = \text{Force} \cdot \frac{\text{Displacement}}{\text{Time}}$$

$$\boxed{\text{Power} = \text{Force} \cdot \text{Velocity}}$$

43. What is meant by product of inertia?

Relative to two rectangular axes, the sum of the products formed by multiplying the mass (or, sometimes, the area) of each element of a figure by the product of the coordinates corresponding to those axes. The values of the products of inertia depend on the orientations of the coordinate axes. For every point of the body or system, there exist at least three mutually perpendicular axes, called the principal axes of inertia, for which the products of inertia are equal to zero.

44. Write short notes on work energy equation.

The Work/Energy Equation is:

$$F_{net}d = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$$

When a net force does work on a rigid body, it causes the body's speed to change.

The work done by the net force is the same as the sum-total of the work done by the action of every force acting the body. If you add up the work done by each of the forces acting on a body you will get the same value as the work done by the net force.

The work done on a body that caused the body to be set in motion with some speed **v** can be expressed as function of the body's final speed **v** and mass **m**, independent of type of force that acted on the body. We call this function the body's Kinetic Energy.

$$KE = \frac{1}{2}mv^2$$

The energy a body possesses by virtue of its motion -- its Kinetic Energy -- is not dependent upon how the object reached its state of motion, only upon its current state of motion.

45. Write short notes on coefficient of restitution.

The coefficient of restitution is the ratio of the particles' relative separation velocity after impact, $(v_B)_2 - (v_A)_2$, to the particles' relative approach velocity before impact, $(v_A)_1 - (v_B)_1$. The coefficient of restitution is also an indicator of the energy lost during the impact. The equation defining the coefficient of restitution, *e*, is

The equation defining the coefficient of restitution, *e*, is

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

46. What is time of restitution, compression and collision?

i. The time taken by the two bodies to regain original shape, after compression is known as time of restitution.

ii. The time taken by the two bodies in compression, after the instant of collision is known as time of compression.

iii. The sum of time of compression and time of restitution is known as time of collision or period of collision or period of impact.

47. Write short notes on Elastic impact.

An elastic collision is an encounter between two bodies in which the total kinetic energy of the two bodies after the encounter is equal to their total kinetic energy before the encounter. Elastic collisions occur only if there is no net conversion of kinetic energy into other forms. In a perfectly elastic collision, no energy is lost and the relative separation velocity equals the relative approach velocity of the particles. In practical situations, this condition cannot be achieved.

48. **State D'Alembert's principle**

The force system consisting of external forces and inertia force can be considered to keep the particle in equilibrium, since the resultant force externally acting on the particle is not zero ,the particle is said to be in dynamic equilibriums. The principle is known as D'Alembert's principle.

$F=ma$

49. Write short notes on collision.

A collision is an event in which two or more bodies exert forces on each other for a relatively short time. Although the most common colloquial use of the word "collision" refers to accidents in which two or more objects collide, the scientific use of the word "collision" implies nothing about the magnitude of the forces.

Some examples of physical interactions that scientists would consider collisions:

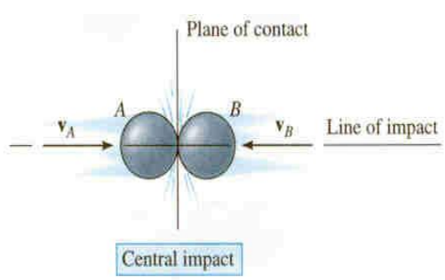
i. An insect touches its antenna to the leaf of a plant. The antenna is said to collide with leaf.

ii. A cat walks delicately through the grass. Each contact that its paws make with the ground is a collision. Each brush of its fur against a blade of grass is a collision. Write short notes on Impact while collision of two body's.

50. Write short notes on Central impact.

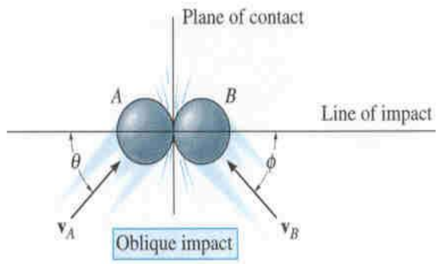
Central impact occurs when the directions of motion of the two colliding particles are along the line of impact.

$(m_A v_A)_1 + (m_B v_B)_1 = (m_A v_A)_2 + (m_B v_B)_2$



51. Write short notes on Oblique impact.

Oblique impact occurs when the direction of motion of one or both of the particles is at an angle to the line of impact.



52. Write short notes on Impact Energy losses.

Once the particles' velocities before and after the collision have been determined, the energy loss during the collision can be calculated on the basis of the difference in the particles' kinetic energy. The energy loss is

$\sum U_{1\text{-}2} = \sum T_2 - \sum T_1$

Where, $T_i = 0.5m_i(v_i)^2$

53. Discus about impact Procedure of analysis.

In most impact problems, the initial velocities of the particles and the coefficient of restitution, *e*, are known, with the final velocities to be determined. Define the x-y axes. Typically, the x-axis is defined along the line of impact and the y-axis is in the plane of contact perpendicular to the x-axis. For both central and oblique impact problems, the following equations apply along the line of impact (x-dir.):

$\boldsymbol{\sum m(v_x)_1 = \sum m(v_x)_2}$

$\boldsymbol{e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]}$

For oblique impact problems, the following equations are also required, applied perpendicular to the line of impact (y-dir.):

$\boldsymbol{m_A(v_{Ay})_1 = m_A(v_{Ay})_2}$

$\boldsymbol{m_B(v_{By})_1 = m_B(v_{By})_2}$

54. Write short notes on the law of conservation of momentum.

Total momentum of an isolated system (no external forces) does not change. Interactions within system do not change the system's total momentum

Isolated system.

$\boldsymbol{m_A(v_{Ay})_1 = m_A(v_{Ay})_2}$

$\boldsymbol{m_B(v_{By})_1 = m_B(v_{By})_2}$

16 mark

1. The driver of an automobile decreases the speed at a constant rate from 72 to 48 km/hr over a distance of 230m along a curve of 460 m radius. Determine the magnitude of the total acceleration of the automobile, after the automobile has travelled 150 m along the curve. (Anna Univ- Nov/Dec 2011) (16)

Solution:

(i) Acceleration needed to clear 230m distance

Equation for normal motion is

$v^2 = u^2 + 2as$

(ie) $\left[\frac{48}{3.6}\right]^2 = \left[\frac{72}{3.6}\right]^2 + 2a \times 230$

$177.78 = 400 + 460a$

Rearranging,

$460a = -400 + 177.78$

$a = -0.483\ m/s^2$

(ii) Velocity needed to cover 150m distance

Here the initial velocity is 72kmph (ie.,20m/s) and distance covered, s=150m

Then,

$v^2 = u^2 + 2as$

$= 20^2 + (2 \times (-0.483) \times 150)$

$v^2 = 400 - 144.93$

$= 255.07$

$v = 15.97\ m/s$

The acceleration 'at' along the tangent of the curve is -0.483 m/s^2 Over a distance of 150m.

(iii) Total acceleration:

Acceleration normal to the curve over a distance of 150m, $a_{n=\frac{v^2}{r}}$ where r is the radius of the curve.

$a_{n=\frac{15.97^2}{460}}$ =0.554 m/s^2

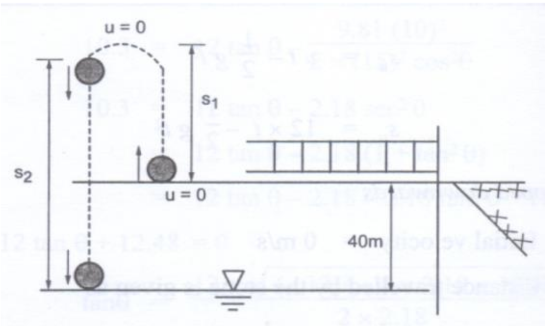
The acceleration, $a = \sqrt{a_n^2 + a_t^2}$

$\sqrt{(0.554)^2 + (-0.483)^2}$

$= 0.735\ m/s^2$

2. A stone is thrown vertically upward from a point on a bridge located 40 m above the water. If it strikes the water 4 sec after release, determine the speed at which the stone was thrown and the speed at which the stone strikes the water. (16)

(Anna Univ- Nov / Dec 2010)



Solution:

Let

S1- distance moved during upward travel
S2-distance by the stone till it strikes the water
V1,V2 respective velocities
(i) Stone movement upwards
Initial velocity is zero,then

$S_1 = ut_1 \ -\frac{1}{2}\, gt_1^2$

$S_1 = 0 \times t_1 \ -\frac{1}{2}\, gt_1^2$

$= \frac{1}{2}\, gt_1^2$

(ii) Stone movement downwards

$S_2 = ut_2 \ -\frac{1}{2}\, gt_2^2$

$S_1 + 40 = 0t_2 \ -\frac{1}{2}\, gt_1^2$

$S_1 + 40 = 4.905\, t_2^2$

$t_1 + t_2 = 4sec$

$t_2 = (4 - t_1)$

Substituting for $s_1 + t_2$ in eqn(ii)

$-\frac{1}{2} \times 9.81\, t_1^2 + 40 = \frac{1}{2} \times 9.81\, (4 - t_1\,)^2$

$-4.905\, t_1^2 + 40 = 4.905\, (16 - 8t_1 - t_1^2)$

$-4.905t_1^2 - 4.905t_1^2 - 78.48 + 39.24t_1 + 40 = 0$

$-9.81t_1^2 + 39.24t_1 - 38.48 = 0$

$9.81t_1^2 - 39.24t_1 + 38.48 = 0$

$t_1 = 39.24 \pm \sqrt{(39.24)^2 - 4 \times 9.81 \times 38.48}\, /(2.9.81)$

$= \frac{39.24 \pm 5.46}{2 \times 9.81}$

$t_1 = 2.28\ or\ 1.72\ sec$

t_1 =2.28 is not valid as the toal travel itself takes 4 sec

Hence $t_1 = 1.72\ sec$, $s_1 = \frac{1}{2} \times 9.81\ 1.72^2 = 14.54m$

$s_2 = 40 + 14.54 = 54.54m$

Speed, $v_1 = \frac{14.54}{1.72} = 8.45\ m/sec$, $v_2 = \frac{54.54}{2.28} = 23.92\ m/sec$

3. **A bomb is dropped from an aeroplane flying at a speed of 80 km/hr at a height of 1500 m above the level ground. Find the horizontal distance covered by the bomb after its release. Also find the time required for the bomb to hit the target and the velocity with which the bomb hits the target. (Anna Univ- Nov / Dec 2010) (16)**

It is assumed that the aeroplane is flying horizondally at the time of release of the bomb.

Thus the velocity of the bomb is initially zero and equals to the speed of aeroplane and moves horizondally later the bomb is attached due to the gravitational effect and moves downwards.

(i) Time equired to hit the target:
Initial velocity of bomb in downward diection,u=0 height of release of bomb,h=1500m
Let t be taken by the bomb to reach the ground
Then,

$$h = ut + \frac{1}{2}gt^2$$

$$1500 = 0 \times t + \frac{1}{2} \times 9.81t^2$$

$$t = \sqrt{\frac{1500}{2 \times 9.81}}$$

$$= 76.45\ sec$$

Time required to reach the ground=76.45 sec
(ii) Horizontal dis tance covered:
Horizontal distance travelled=(horizontal velocity of the bomb × time

$$\frac{800 \times 1000}{3600} \times 76.45$$

$$= 222.22m$$

Horizontal distance covered = 222.22m

4. Two electric trains A and B leave the same station on parallel lines. Train A starts with a uniform acceleration of 0.15 m/s² and attains the speed of 40 km/hr, when the steam is reduced to keep the speed constant. Train B leaves 1 min after with a uniform acceleration of 0.3 m/s² to attain the max speed of 70 km/hr. When the train B will over take the train A. (Anna Univ- DEC 2014) (16)

Solution:
(i) considering the movement of train A:
From the initial of zero, train A attain a speed of 40 km/hr and lets off steam to maintain a uniform speed.
We know, $v = u + at$

$$at = v - u$$

$$t_A = \frac{v-u}{a} \quad \left(v_A = \frac{40 \times 1000}{3600} = 11.11m/sec\right)$$

$$= \frac{11.11 - 0}{0.15} = 74.1\ sec$$

Distance travelled by train A during 74.1sec, S1= $ut + \frac{1}{2}at^2$

$$= 0 + \frac{1}{2} \times 0.15 \times 74.1^2 == 411.81m$$

Total distance travelled by train A beyond t=74.1 sec

$$= 411.81 + 11.11(t - 74.1)$$

$$S_2 = 11.11t - 411.44 ------------ (i)$$

(2) Consider the movement of train B

Here $v_B = \frac{70\times1000}{3600} = 19.44\ m/sec$

Train B states 60sec, after starting of train A time taken by train B to attain a speed of 19.44 m/sec $t_B = \left(\frac{v-u}{a}\right) + 60$

$= \left[\frac{19.44 - 0}{0.3}\right] + 60 = 64.8 + 60 = 124.8\ sec$

Distance travelled by train B in 60 sec=$ut + \frac{1}{2}at^2$

$S_3 = 0 + \frac{1}{2} \times 0.3 \times 60^2 = 540m$

Total distance covered by train B beyond 24.8 sec

$S_4 = 540 + 19.44(t - 124.8)$

$= 19.44t - 1886.1 - - - - - - - - - - (ii)$

At the instant of train B overtaking train A equation (i)+(ii) are equal

Ie., 11.11t-411.44=19.44t-1886.1

19.44-11.11t=1886.1-411.11

8.33t=1474.66

t=177.03 sec

train B will overtake when the time is 177.03 sec from starting.

5. A stone is projected with the speed of 30 m/s at an angle of elevation of 50º. Find its velocity (i) After 2 sec (ii) At the highest point of its point (iii) At a height of 6 m.Find also the time interval between the two points at which the stone attains a speed of 23 m/s. (Anna Univ-Apr/May 2006) (16)

Solution:

Stone is projected at 50° angle of elevation

Hence initial horizontal velocity , u=30 sin 50° =22.98 m/sec

(i) Velocity after 2 secs:

$v = u - gt$

$= 22.98 - 9.81 \times 2 = 3.36\frac{m}{sec}$

(ii) Velocity at the highest point of its path:

At the highest point $v_y = 0$

Then $v = v_x = 30cos50° = 19.28\ \mathrm{m/sec}$

(iii) Velocity of a height of 6m:

This is obtained from an expression

$v = v^2 - 2gh$

$= 22.98^2 - 2 \times 9.81 \times 6 = 20.26\ m/sec$

(iv) Time travelled b/w two points at which the speed is 23 m/s:

The height 'h' atwhich the stone attains a speed of 23m/sec is got from the expression assuming angle of projection as 50° at this point.

$v^2 = u^2 - 2gh$

$(23 \sin 50°)^2 = 22.98^2 - 2 \times 9.81 \times h$

$$h = \frac{22.98^2 - 17.62^2}{2 \times 9.81} = h = 11.09m$$

vertical distance ,$h = usin \propto t - \frac{1}{2}gt^2$

ie., $gt^2 - 2ausin \propto t + 2h = 0$

$9.81t^2 - 2 \times 22.98 \times t + 2 \times 11.09 = 0$

$9.81t^2 - 45.96t + 22.18 = 0$

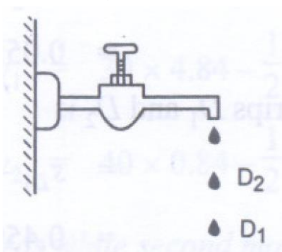
$$t = \frac{45.96 \pm \sqrt{45.96^2 - 4 \times 9.81 \times 22.18}}{2 \times 9.81}$$

$$= \frac{45.96 \pm 35.24}{19.62}$$

$t = 4.14 \sec(or)\ 0.55\ sec$

Time difference = 4.14-0.55=3.59 sec.

6. Water drops from a faucet at the rate of 5 drops per sec as shown in fig. Determine the vertical separation between two consecutive drops after the lower drop has attained a velocity of 3 m/s. (Anna Univ- Nov / Dec 2004) (16)



Solution:

Rate of drop is 5 drops per second

Ie., each drop falls after 1/5 of a second

Let D1 be the first drop cameout after 1/5 of a second followed by the drop D2.

(i) Considering motion of drop D1:

` velocity of lower drop, D1=3 m/s

Using the expression

$v = u + gt$

$3 = 0 + 9.81t \quad (initial\ velocity, u = 0)$

$$t = \frac{3}{9.81} = 0.306\ sec$$

Vertical distance moved by drop D1 in 0.306 sec is

$$S_{D1} = ut + \frac{1}{2}gt^2$$

$$= 0 \times t + \frac{1}{2} \times 9.81 \times 0.306^2 = 0.459\ m$$

(ii) Considering motion of drop D2:

The upper drop D2 will take $(0.306 - \frac{1}{5})$ sec

0.306-0.20=0.106 sec

Vertical distance moved by drop D1 in time 0.106 sec is given as,

$$S_{D2} = ut + \frac{1}{2}gt^2$$

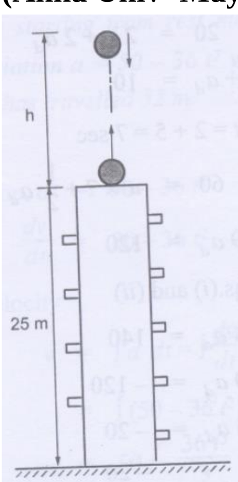
$$= 0 \times t + \frac{1}{2} \times 9.81 \times 0.106^2 = 0.055\,m$$

Vertical separation between drops D1 and D2 is

$$= S_{D1} - S_{D2}$$

$$= 0.459 - 0.055 = 0.404\,m$$

7. A steel ball is thrown vertically upwards from the top of a building 25 m above the ground with an initial velocity of 18 m/s. Find the maximum height reached by the ball from the ground. (Anna Univ- May/June 2003) (16)



Solution:

Using the expression,

$$v^2 = u^2 - 2gh$$

At the highest point the velocity is zero,then $v = 0$

$$0 = 18^2 - 2 \times 9.81 \times h$$

$h = \frac{18^2}{2 \times 9.81} = 16.51m$

Max height reached by the ball from the ground

=height of the building +h

$= 25 + 16.51 = 41.51m$

Max height reached by the ball from the ground=41.51m.

8. A particle under constant deceleration is moving in a straight line and covers a distance of 20 m in the first 2 sec and 40 m in the next 5 sec and calculate the distance it covers in the subsequent 3 sec and total distance travelled by the particle before it comes to rest. (Anna Univ-Nov / Dec 2003) (16)

Solution :

Acceleration, $a = 50 - 36t^2$

$\frac{dv}{dt} = 50 - 36t^2$

$v = \int adt = \int \frac{dv}{dt} \quad = \int (50 - 36t^2)dt$

$= 50t - \frac{36t3}{3} + c1 ------(ii)$

As the particle has started from rest,

$t = 0, v = 0$

$c_1 = 0, ie., v = 50t - 12t^3 -----(iii)$

Displacement, $s = \int vdt$

$= \int (50t - 12t^3)dt = \frac{50t^2}{2} - \frac{12t^4}{4} + c2$

$S = 25t^2 - 3t^4 + c2$

As the particle has moved from rest, t=0,s=0 and hence c2=0,

$S = 25t^2 - 3t^4$

$52 = 25t^2 - 3t^4$

$t^2 = \frac{25 \pm \sqrt{(-25)^2 - 4 \times 3 \times 52}}{2 \times 3}$

$= \frac{25 \pm 1}{6}$

$t = \sqrt{\frac{25}{6}} = \sqrt{4} = 2.0\sec$

Then velocity at 2 seconds, $v^2 = 50 \times 2 - 12 \times 2^3$ =4m/sec.

9. **A particle starting from rest moves in a straight line and its acceleration is given by the relation a = 50 – 36 t^2 where t is in sec. Determine the velocity of the particle when it has travelled 52 m. (Anna Univ- Jan 2013) (16)**

Solution:

Let ad be the deceleration, then

Displacement, $S = ut + \frac{1}{2}a_d t^2$

During 1st phase, t=2 sec ,then $20 = u \times 2 + \frac{1}{2}a_d 2^2$

$= 2u + 2a_d$

$u + a_d = 10 ------ (i)$

During the just two phases, t=2+5=7 sec

$60 = u \times 7 + \frac{1}{2}a_d 7^2$

$14u + 49a_d = 120 ----- (ii)$

Solving for u and ad from eqn (i)+(ii)

$14 \times eqn(i) \quad 14u + 14a_d = 140$

$- \times eqn(ii) \quad -14u - 49a_d = -120$

$35a_d = -20$, $a_d = \frac{-20}{35} = -0.571 m/s^2$

$x = 10 + 0.51 = 10.571 m/\sec$

During the third phase,

T=7+3=10 sec

$S = 10.57 \times 10 - \frac{1}{2} \times 0.571 \times 10^2 = 77.15m$

Distance covered in subsequent 3 sec=77.15-60=17.15 m

During fourth phase when it comes to rest

Velocity at t=10 sec, $v = u + at$

$= 10.57 - 0.571 \times 10 = 4.857 m/\sec$

At rest, v=0

$0 = 4.857 - 0.571 \times t$

$t = \frac{4.857}{0.571} = 8.51 \sec$, total time =2+5+3+8.51=18.51 sec

Total distance travelled, $S = 10.57 \times 18.57 - \frac{1}{2} \times 0.571 \times 18.51^2 = 97.83m$

10. **The position of the particle which moves along the straight line is defined as s = t^3- 6t^2 - 15t + 40 where s is expressed in m and t in sec. Determine (a) time at which the velocity will be zero, (b) the position and distance travelled by the particle at that time, (c) the acceleration of the particle at that time, (d) the distance travelled by the particle when t = 4 to t = 6 sec. (Anna Univ- Nov / Dec 2002) (16)**

Solution:

(a)time at which velocity is zero

$S = t^3 - 6t^2 - 15t + 40$

Velocity, $v = \frac{ds}{dt} = 3t^2 - 12t - 15$

$3t^2 - 12t - 15 = 0$, $t^2 - 4t - 5 = 0$, $(t-5)(t+1) = 0$, $t = 5(or) - 1\sec$

T=-1 is impossible, t=5 sec

Ie., the time at which velocity will be zero is 5sec

(b) position and distance travelled:

At t=5 sec, $S = 5^3 - 6\times5^2 - 15\times5 + 40 = -60m$

Considering scalar quantity only the position is at 60 m

When t=0,S=40m

Distance travelled = -60-40=-100m

Ie., the travelled is 100 m considering only the scalar quantity,

(c) Accelaration of the particle

$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 12t - 15)$

$a = 6t - 12$

When t=5sec, $a = 6\times5 - 12 = 18m/\sec^2$

(d) distance travelled from t=4 to t=6 sec

$S = t^3 - 6t^2 - 15t + 40$

When t=4 sec, $S = 4^3 - 6\times4^2 - 15\times4 + 40 = -52m$

When t=6sec, $S = 6^3 - 6\times6^2 - 15\times6 + 40 = -50m$

Distance travelled from 4 to 5 sec = -60-(-52)=-8 m

Distance travelled from 5 to 6 sec = -50-(-60) = 10m

Total distance travelled from 4 to 6 sec

=8+10= 18m.

11. The motion of the particle is described by an equation, displacement s = 5t^2 - 7t + 2. Find (a) displacement, velocity, acceleration when time t = 2 sec, (b) minimum displacement and corresponding velocity and acceleration. Take s in m. (16)

(Anna Univ- Nov / Dec 2001)

Solution:

Displacement, $S = 5t^2 - 7t + 2$

Velocity, $v = \frac{ds}{dt} = \frac{d}{dt}(5t2 - 7t + 12)$

$v = (10t - 7)m/\sec$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(10t - 7) = 10m/s^2$

(a) Displacement , velocity and acceleration:

When t=2 sec

Displacement, $S = 5\times2^2 - 7\times2 + 2 = 8m$

Velocity, $v = 10\times2 - 7 = 13m/\sec$

Acceleration is constant at all times ie., a=10 m/s^2

(b) Minimum displacement, velocity and acceleration:

For minimum displacement, $\frac{ds}{dt} = 0$

$v = 0, 10t - 7 = 0, t = \frac{7}{10} = 0.7\,\text{sec}$

Then displace ment, $S = 5(0.7)^2 - 7 \times (0.7) + 2 = -0.45m$

Acceleration is constant, ie., a=10m/s2

Result:

(a) When t=2 sec

Displacement=S=8m

Velocity=v=13m/s

Acceleration=a=10m/s

(b) Minimum values

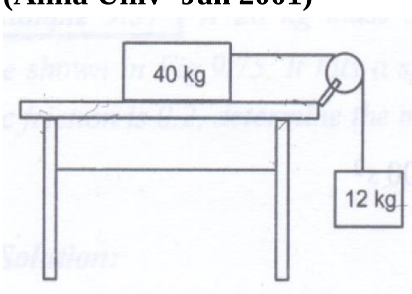
S=-0.45m

V=0

A=10m/s2

12. A 40 kg mass is dragged along the surface of a table by means of a cord which passes over a frictionless pulley at the edge of the table and is attached to a 12 kg mass. If the coefficient of friction between the 40 kg mass and the table is 0.15, determine the acceleration of the system and the tension in the cord. (16)

(Anna Univ- Jan 2001)



SOLUTION:

Free body diagram for 12kg block and 40kg block are shown in fig

(i) Consideriong 12 kg block:

As te 12 kg block tents to move downwards, the inertia force m2a acts upwards

Let T be the tension in the chorde

Considering the forces vertically,

$$T = m_2 a = m_2 g$$
$$T = 12 \times 9.81 - 12a$$
$$T = 117.7 - 12a ------(i)$$

(ii) Considering 40kg block:

Because of the weight of 12kg block and frictionless pulley the body moves down. Hence the inertia forces acts left. Further the frictional pulleys force 'F' acts towards left.

Resolving the forces horizontally,

$T_1 - m_1 a - F = 0$

Ie., $T = 40a + (0.15 \times 40 \times 9.81) - - - (\because F = 0 \sin cesmoothplane)$

$T = 58.86 + 40a - - - - - - - - (ii)$

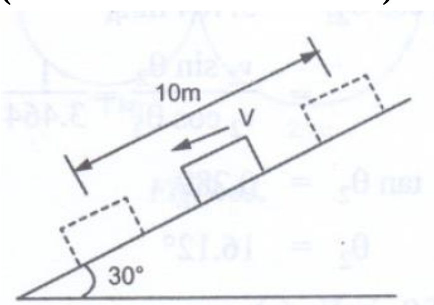
Equating eqn (i) and (ii)

$117.7 - 12a = 58.86 + 40a$

$52a = 117.7$

$a = 58.86$

13. If a 70 kg block shown in fig is released from rest at A. Determine its velocity after it slides 10 m down the plane. Take the coefficient of friction as 0.30. (16)
(Anna Univ- Nov / Dec 2011)



Solution:

Using the relationship

$$W \sin \theta - F_R)s = \frac{1}{2}\frac{W}{g}({v_2}^2 - {v_1}^2)$$

$$(W \sin 30 - FR)10 = \frac{1}{2}\frac{W}{9.81}({v_2}^2 - 0)$$

$$(W \sin 30 - pW \cos 30)10 = \frac{{v_2}^2}{19.62}$$

$$(0.5 - 0.3 \times 0.866)10 = \frac{{v_2}^2}{19.62}$$

${v_2}^2 = 19.62 \times 2.402$

$= 47.13$

$velocity, v = 6.87 m/\sec$

14. **A lift has an upward acceleration of 1 m/s^2. Calculate the reaction that a man of mass 65 kg will produce on the floor of the lift. How will this reaction change if the lift moves with a downward acceleration of 3 m/s^2? Also find an upward acceleration which will produce a reaction of 767 N. (Anna Univ- Nov / Dec 2011) (16)**

(i) upward movement:

Weight on lift due to one man= $65 \times 9.81 = 637.5N$

Reaction on the lift of the floor, $T_1 = W(1 + \frac{a_1}{g})$

$= 637.65(1 + \frac{1}{9.81}) = 702.65N$

(ii) downward movement:

Reaction on the lift of the floor, $T_2 = W(1 - \frac{a_2}{g})$

$= 637.65(1 - \frac{3}{9.81}) = 442.65N$

(iii) upward acceleration:

Reaction on the lift of the floor, $T_3 = W(1 + \frac{a_3}{g})$

$767 = 637.65(1 + \frac{a_3}{9.81})$

$1 + \frac{a_3}{9.81} = 1.203$

$a_3 = (1.203 - 1) \times 9.81 = 1.99 m/\sec^2$

15. A 100 gm cricket ball has a velocity of 20 m/s before being struck by the bat. After the impact the ball moves in the direction as shown in the fig. with a velocity of 30 m/s. If the ball and the bat are in contact for 0.01 sec, determine the average impulse force exerted on the ball during the impact. (Anna Univ- Nov / Dec 2007 & 2011) (16)

Solution:

The ball tits the bat horizontally and then directed by 30

Hence, the initial horizontal velicity

$u_x = -20 m/s$

$u_y = 0$

After titting the velocity of the ball are

$v_x = 30\cos 30^\circ = 25.98 m/\sec$

$v_y = 30\sin 30^\circ = 15 m/\sec$

Applying impulse – momentam principle in x direction, we have

$\sum F_x t = m(v_x - u_x)$

$F_x \times 0.01 = \frac{100}{1000}(25.98 - (-20))$

$F_x = \frac{0.10}{0.01}(45.98) = 459.8$

Similarly in y-direction

$$\sum F_y t = m(v_y - u_y)$$

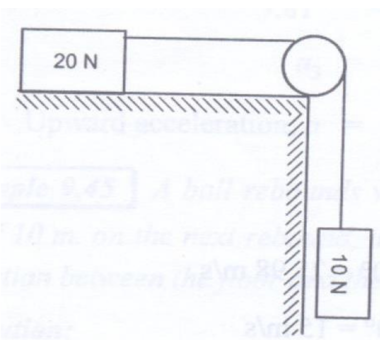
$$Fy \times 0.01 = 0.1(15-0)$$

$$Fy = \frac{0.1}{0.01}\times 15 = 150N$$

$$F = \sqrt{F_x^{\ 2} + F_y^{\ 2}}\ , \quad F = 483.64N$$

.

16. Two bodies of weight 20 N and 10 N are connected to the two ends of a light inextensible string, passing over a smooth pulley. The weight of 20 N is placed on a horizontal surface with the weight of 10 N is hanging free in air as shown in fig. The horizontal surface is rough one, having coefficient of friction between the weight 20 N and plane surface equal to 0.3. Using Newton's second law of motion, determine (i) the acceleration of the system and (ii) tension in the string. (Anna Univ- Apr/May 2005)
(16)



Solution:

Let T be the tension in the string. The free body diagram is drawn as shown in fig.

(i) Considsering the 10N weight :

This weight will be moving downwards causing the 20N to move to the right force acting on 10N weight along with inertia force as shown in fig.

Resolving the forces vertically,

$$T - 10 + ma = 0$$

$$T - 10 + \frac{10}{9.81}a = 0$$

$$T \pm 1.02a = 10 - - - - - (i)$$

(ii) Considering 20N weight:

Here R is the reaction and FR is the frictional resistance. All the forces acting on the 20N weight along with the inertia force are shown in fig. as the body moves to the right the inertia force and frictional resistance act in the opposite direction.

Resolving the forces horizontally,

$$T - F_R - ma = 0$$

$$T - pR - \frac{W}{g}a = 0$$

$$T - 0.3 \times 20 - \frac{20}{9.81}a = 0$$

$$-2.039a + T = 6 ------(ii)$$

Subtracting Eqn (ii) from Eqn (i)

$$1.02a + 2.039a = 10 - 6$$

$$a = \frac{4}{3.059} = 1.308 m/s^2$$

$$T = 6 + 2.039 \times 1.308$$

$$= 8.667N$$

17. Two bodies weighing 300 N and 450 N are hung to the ends of rope passing over an ideal pulley. With what acceleration the heavier body comes down? What is the tension in the rope? (Anna Univ- Nov / Dec 2002) (16)

Solution:

Free body diagram is shown in fig

(i) Considering Block A(moving down wards)

Forces acting on block A along with the inertia force are shown in fig.

$$T - 450 + m_A a = 0$$

$$T - 450 + \frac{450}{9.81}a = 0$$

$$ie., T = 45.87a = 450 -----(i)$$

(ii) Considering block B: (moving upwards)

Forces acting on block B along with the inertia force are shown in fig.

$$T - 300 + m_B a = 0$$

$$T - \frac{300}{9.81}a = 300$$

$$T - 30.58a = 300 -----(ii)$$

Subtracting eqn (ii) from eqn (i)

$$45.87a + 30.58a = 450 - 300$$

$$76.45a = 150$$

$$a = 1.96 m/s^2$$

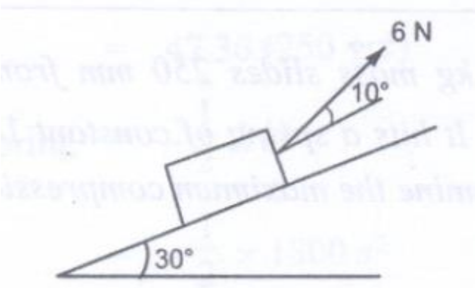
Sub a in eqn (i)

$$T + 45.87 \times 1.96 = 450$$

Tension in rope, $T = 360.1N$

18. A mass of 1 kg is pulled up by a smooth plane by a 6 N force, acting at an angle of 10 º to the plane surface, as shown in fig. If the mass is not loosing contact with the plane, Determine (i)

Normal reaction offered by the plane on the mass (ii) the acceleration of the mass along the plane. (Anna Univ- Nov / Dec 2001) (16)



Solution:

By resolving the forces along the plane,

$P\cos\theta = ma + W\sin\alpha$

$6\times\cos 10^\circ = 1\times a + 1\times 9.81\times\sin 30^\circ$

$5.909 = a + 9.81\times 0.5$

$a = 1.00 m/s^2$

By resolving the forces perpendicular to the plane.

$P\sin\theta + R = W\cos\alpha$

$6\sin 10^\circ + R = 1\times 9.81\times\cos 30^\circ$

$1.042 + R = 8.496$

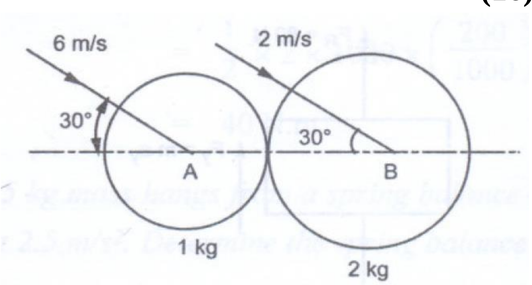
$R = 7.45N$

Normal reaction offered by the plane on the mass, 'R'

$R = 7.45N$

Accelaration of the mass along the plane, $a = 1.00 m/s^2$

19. A ball of mass 1 kg moving with a velocity of 6 m/s strikes another ball of mass 2 kg moving with a velocity of 2 m/s at the instant of impact. The velocities of the two balls are parallel and inclined at 30 ° to the line of joining their centres as shown in fig. If the coefficient of restitution is 0.5, find the velocities and the directions of the two balls after impact. (Anna Univ- May/June 2010, Anna Univ- Nov / Dec 2001, Anna Univ- May/June 2012) (16)



Solution:

The verticalcomponents of velocities before and after impact are the same.

$u_1\sin\alpha_1 = v_1\sin\theta_1$

$u_2 \sin \alpha_2 = v_2 \sin \theta_2$

But $u_1 \sin \alpha_1 = 6 \times \sin 30^\circ = 3m/s$

$ie., v_1 \sin \theta_1 = 3m/s ------(i)$

Apply $u_2 \sin \alpha_2 = 2 \sin 30^\circ = 1m/s$

$v_2 \sin \theta_2 = 1m/s -------(ii)$

Applying law of conservation of momentum along the line of impact

$m_1 u_1 \cos \alpha_1 + m_2 u_2 \cos \alpha_2 = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2$

$1 \times 6 \times \cos 30^\circ + 2 \times 2 \times \cos 30^\circ = (1 \times v_1 \cos \theta_1) + (2 \times v_2 \cos \theta_2)$

$8.66 = v_1 \cos \theta_1 + 2v_2 \cos \theta_2 -------(iii)$

Applying Newtons law of collision along the line of impact:

$v_2 \cos \theta_2 - v_1 \cos \theta_1 = e(u_1 \cos \alpha - u_2 \cos \alpha_2)$

$= 0.50(6 \cos 30^\circ - 2 \cos 30^\circ)$

$v_2 \cos \theta_2 - v_1 \cos \theta_1 = 1.732 -------(iv)$

Solving eqn (iii) & (iv)

$v_1 \cos \theta_1 + 2v_2 \cos \theta_2 = 8.66$

$-v_1 \cos \theta_1 + v_2 \cos \theta_2 = 1.732$

$3v_2 \cos \theta_2 = 10.392$

$v_2 \cos \theta_2 = 3.464 m/\sec ------(v)$

Dividing eqn (ii) by eqn (v)

$$= \frac{v_2 \sin \theta_2}{v_2 \cos \theta_2} = \frac{1}{3.464}$$

$\tan \theta_2 = 0.289$, $\theta_2 = 16.12$

Sub θ_2 in eqn (v)

$v_2 \cos \theta_2 = 3.464$

$$v_2 = \frac{3.464}{\cos 10.12} = 3.61 m/s$$

From eqn (iv)

$-v_1 \cos \theta_1 + v_2 \cos \theta_2 = 1.732$

$-v_1 \cos \theta_1 + 3.464 = 1.732$, $v_1 \cos \theta_1 = 1.732 -------(vi)$

Devide eqn (i) by (vi)

$$\frac{v_1 \sin \theta_1}{v_1 \cos \theta_1} = \frac{3}{1.732} = 1.732$$

$\tan \theta_1 = 1.732 \qquad \theta_1 = \tan^{-1}(1.732) = 60^\circ$

Sub θ_1 in eqn (vi)

$v_1 \cos \theta_1 = 1.732$

$v_1 = 3.464 m/\sec$.

20. An object is thrown vertically upwards with a velocity of 30 m/s. Four seconds later a second object is projected vertically upwards with a velocity of 40 m/s. Determine (i) the time (after the first object is thrown) when the two object will meet each other in air, (ii) the height from the earth at which the two objects will meet. (16)

(Anna Univ- Nov / Dec 2004)

Solution:

Possibilities of meeting when both the objects (i) moving upwards (ii) when the the just object returning are to be tried.

Let they meet 'y' metre above the ground.

(i) Both the objects moving upwards:

$$y_1 = u_1t_1 - \frac{1}{2}gt_1^2 \text{ and } y_2 = u_2t_2 - \frac{1}{2}gt_2^2$$

Let, $y_1u_1 + t_1$ -correspond to first object and

$y_2u_2 + t_2$-correspond to first object respectively.

At the time of meeting y1=y2

$$u_1t_1 - \frac{1}{2}gt1_2 = u_2t_2 - \frac{1}{2}gt_2^2$$

$$30 \times t_1 - \frac{1}{2} \times gt1_2 = 40(t_1 - 4) - \frac{1}{2}g(t_1 - 4)^2 \; (\because t_2 = t_1 - 4)$$

$$30 \times t_1 - \frac{1}{2} \times 9.81t_1^2 = 40t_1 - 160 - \frac{1}{2} \times 9.81(t_1^2 - 8t_1 + 16)$$

$$30 \times t_1 - 4.905t_1^2 = 40t_1 - 160 - 4.905t_1^2 + 39.24t_1 - 78.48$$

$$49.24t_1 = 238.48$$

$$t_1 = 4.84 \text{sec}$$

$$\text{t}_2 = 4.84 - 4.0 = 0.84 \text{sec}$$

$$y_1 = 30 \times 4.84 - \frac{1}{2} \times 9.81 \times 4.84^2 = 30.20m$$

$$y_2 = 40 \times 4.84 - \frac{1}{2} \times 9.81 \times 0.84^2 = 30.14m$$

(ii) First object moves down wards while second moves upwards:

For object,(i)

$$v_1 = u_1 + at_1$$

$$0 = 30 - 9.81 \times t_1$$

$$t_1 = \frac{30}{9.81} = 3.058 \text{sec} \text{ is the time taken to go up}$$

$$y_1 = 0 + \frac{1}{2} \times 9.81 \times 3.058^2 = 45.87m$$

For object (2)

$$t_2 = t_1 + (4 - 3.058) = t_1 - 0.942$$

Now $y_1 = 45.87 - \frac{1}{2} \times 9.81 \times t_1{}^2$

$$y_1 = 45.87 - 4.905 t_1{}^2 \; ------ (iii)$$

For object (2)

$$y_2 = 40(t1 - 0.942) - \frac{1}{2} \times 9.81 \times (t_1 - 0.942)^2$$

$$= 40(t1 - 0.942) - 4.905(t_1 - 0.942)^2 \; ------- (iv)$$

Now

y1=y2

$$45.87 - 4.905 t_1{}^2 = 40(t1 - 0.942) - 4.905(t_1 - 0.942)^2$$

$$45.87 - 4.905 t_1{}^2 - 40 t_1 + 37.68 + 4.905(t_1{}^2 - 1.884 t_1 + 0.887) = 0$$

$-49.22 t_1 + 87.91 = 0$, $\quad t_1 = 1.786 \sec$

Now , $y_2 = 40(1.786 - 0.94) - 4.905(1.786 - 0.942)^2$

Total time taken for the object to meet = (3.058+1.786) , =4.844 sec

Height of thew ball at time of meet=30.26m